> Workbook answers

Exercise 1.1

1　**a**
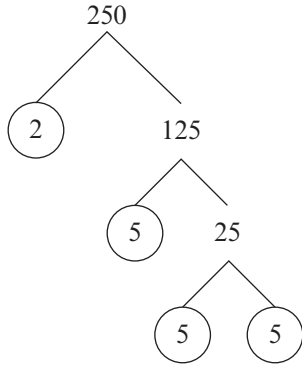
　　b　No. The 125 can only become 5×25 and 25 as a factor of primes must be 5×5.

　　c
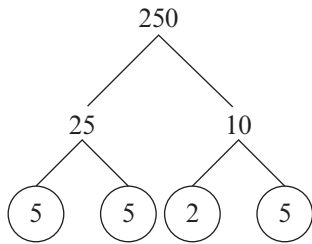
　　d　$250 = 2 \times 5^3$

2　**a & b** Many trees are possible but all end with 2, 2, 3, 5, 5.

　　c　$300 = 2^2 \times 3 \times 5^2$

3　**a**　**i**　2×3　　　　**ii**　$2 \times 3 \times 5$

　　　　iii　$2 \times 3 \times 5 \times 7$

　　b　$2 \times 3 \times 5 \times 7 \times 11 = 2310$; multiply the last number by the next prime

4　**a**　42

　　b　1764

　　c　74 088

5　**a**　Many trees are possible

　　b　$8712 = 2^3 \times 3^2 \times 11^2$

6　**a**　$96 = 2^5 \times 3$

　　b　97 is a prime number

　　c　$98 = 2 \times 7^2$

　　d　$99 = 3^2 \times 11$

7　**a**　$70 = 2 \times 5 \times 7$

　　b　$70^2 = 2^2 \times 5^2 \times 7^2$

　　c　$70^3 = 2^3 \times 5^3 \times 7^3$

8　**a**　**i**　3^2　　　　**ii**　$2^2 \times 3^2$

　　　　iii　3^4　　　　**iv**　$2^4 \times 3^2$

　　　　v　$3^2 \times 5^2$　　**vi**　$2^6 \times 3^2$

　　　　vii　5^4　　　　**viii**　7^4

　　b　There is an even number of each prime factor.

　　c　Using the result of part b, it is the square of $2^2 \times 3 \times 5 \times 7$.

9　**a**　$3^2 \times 7 = 63$

　　b　$3 \times 5 = 15$

　　c　$2^2 \times 3 = 12$

10　**a**　360　　**b**　300　　　**c**　1800

11　**a**　$104 = 2^3 \times 13$

　　b　$130 = 2 \times 5 \times 13$

　　c　26

　　d　520

12　**a**　$135 = 3^3 \times 5$

　　b　$180 = 2^2 \times 3^2 \times 5$

　　c　45

　　d　540

13　**a**　$343 = 7^3$

　　b　$546 = 2 \times 3 \times 7 \times 13$

　　c　7

　　d　26 754

14　630

15　**a**　24　　**b**　1848

16　**a**　$48 = 2^4 \times 3$ and $25 = 5^2$; there are no common prime factors, therefore the LCM is 1.

　　b　1200

17　18 and 24

Exercise 1.2

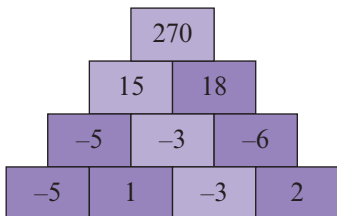**1** $-1 \times -4 = 4$; $-3 \times -4 = 12$; $-5 \times -4 = 20$

2 a -40 **b** 40 **c** 99 **d** 120

3 A, B, D, F in one group and C, E in the other

4

×	2	−4	−9
−6	−12	24	54
5	10	−20	−45
−8	−16	32	72

5 a 35 **b** -5 **c** 35 **d** 5

6 a 24 **b** -66 **c** 81 **d** 16

7 $(-6)^2 + (-8)^2 - (-10)^2 = 36 + 64 - 100 = 0$

8 a

```
            96
        −12    −8
      −6    2    −4
    3    −2    −1    4
```
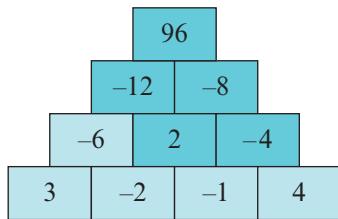
b If 3 and −2 are swapped and −1 and 4 are swapped, then the top number will be 3456.

9 a 1×-6 or -1×6 or 2×-3 or -2×3

b 1×6 or -1×-6 or 2×3 or -2×-3

10 a $63 \div -9 = -7$ or $63 \div -7 = -9$

b $-84 \div 12 = -7$ or $-84 \div -7 = 12$

11 a -6 **b** 5 **c** -9 **d** 13

e -12

12 a -3 **b** 2 **c** -8 **d** -4

13

```
            270
        15      18
      −5    −3    −6
    −5    1    −3    2
```

14 a -6 **b** 12 **c** -12 **d** 8

15 a 32 **b** -40 **c** -4 **d** -5

16 a True. $-3 \times (-6 \times -4) = -3 \times 24 = -72$ and $(-3 \times -6) \times -4 = 18 \times -4 = -72$

b False. $-24 \div (-4 \div -2) = -24 \div 2 = -12$ and $(-24 \div -4) \div -2 = 6 \div -2 = -3$

Exercise 1.3

1 a 196 **b** 196 **c** 400 **d** 900

2 a 64 **b** -216 **c** -1000 **d** 0

3 a impossible **b** -4

c -5 **d** -9

4 a $x = 5$ or -5 **b** $x = 15$ or -15

c $x = 9$ or -9 **d** no solution

5 a $x = 6$ **b** $x = -6$

c $x = -10$ **d** $x = -20$

6 a $x = 23$ or -23 **b** no solution

c $x = 23$ **d** $x = -23$

7 a true **b** false **c** true

d true **e** true

8 a

x	−3	−2	−1	0	1	2
$x^2 + x$	6	2	0	0	2	6
$x^3 + x$	−30	−10	−2	0	2	10

b i $x = -2$ or 1

ii $x = 1$

9 a Yes. If $x = 5$ then $x^3 - x = 5^3 - 5 = 125 - 5 = 120$

b No. If $x = -5$ then $x^3 - x = -125 - -5 = -120$

10 a $64 = 2^6$

b $2^6 = (2^3)^2 = 8^2$ and $(2^2)^3 = 4^3$

c $729 = 3^6$

d $3^6 = (3^3)^2 = 27^2$ and $(3^2)^3 = 9^3$

e 1 is both a square number and a cube number. So is $4^6 = 4096$ or $5^6 = 15\,625$; other answers are possible.

11 $x^6 = 64$

So $(x^3)^2 = 64$

So $x^3 = 8$ or -8

If $x^3 = 8$ then $x = 2$

If $x^3 = -8$ then $x = -2$

There are two possible answers, $x = 2$ or -2

Exercise 1.4

1 **a** 3^3 **b** 7^4 **c** 12^6 **d** 15^5

2 **a** 6^6 **b** 10^7 **c** 3^9 **d** 14^7

3 **a** $2^0 + 2^1 + 2^2 + 2^3 = 1 + 2 + 4 + 8 = 15 = 16 - 1 = 2^4 - 1$

   b $2^6 - 1$

   c No. $3^0 + 3^1 + 3^2 + 3^3 = 1 + 3 + 9 + 27 = 40$ and $3^4 - 1 = 81 - 1 = 80$ so they are not equal.

4 **a** 5^6 **b** 15^6 **c** 7^9 **d** 3^{20}

5 **a** 2^2 **b** 2^6 **c** 3^6

6 **a** 5^8 **b** 5^{12} **c** 5^{16}

7 **a** 4^3 **b** 7^2

   c 15^3 **d** 15^0 or 1

8 **a** 8^2 **b** 5^4 **c** 2^8 **d** 3^3

   e 12^0 or 1

9 **a** 6^3 **b** 6^4 **c** 6^8 **d** 6^6

10 **a** 2^7 **b** 3^3

   c 2^4 or 4^2 **d** 3^0 or 1

11 **a** 5^3 **b** 5^6 **c** 5^{12}

12 **a** 12^8 **b** 12^{12} **c** 12^2

13 No, Marcus is not correct.

   $2^4 = 2 \times 2 \times 2 \times 2 = 16$ and $4^2 = 4 \times 4 = 16$ so these are equal.

   However $3^4 = 3 \times 3 \times 3 \times 3 = 81$ and $4^3 = 4 \times 4 \times 4 = 64$ and these are not equal.

Exercise 2.1

1 **A** and **ii**, **B** and **vi**, **C** and **v**, **D** and **iii**, **E** and **iv**, **F** and **i**

2 **a** 3 books: $3 \times 2 = 6$

   b 5 books: $5 \times 2 = 10$

   c 8 books: $8 \times 2 = 16$

   d x books: $x \times 2 = 2x$

   e y books: $y \times 2 = 2y$

   f b books: $b \times 2 = 2b$

3 **a** 4 sweets: $4 \div 2 = 2$

   b 10 sweets: $10 \div 2 = 5$

   c 12 sweets: $12 \div 2 = 6$

   d x sweets: $x \div 2 = \dfrac{x}{2}$

   e y sweets: $y \div 2 = \dfrac{y}{2}$

   f s sweets: $s \div 2 = \dfrac{s}{2}$

4 **a** $c - 2$ **b** $c + 2$

   c $\dfrac{c}{2}$ **d** $2c$

5 **A** and **v**, **B** and **i**, **C** and **vi**, **D** and **ii**, **E** and **iv**, **F** and **iii**

6 **a** $7n + 4$ **b** $\dfrac{n}{6} - 8$

   c $\dfrac{n + 4}{5}$ **d** $\dfrac{n - 4}{5}$

7 **a** Equivalent to $\dfrac{7x}{8}$ are: **A, E, F, G, J**

   Equivalent to $\dfrac{x + 7}{8}$ are: **D, I**

   Equivalent to $x + \dfrac{7}{8}$ are: **C, H**

   b **B** $\dfrac{x - 7}{8}$

8 The answer to a is incorrect. It should be $\dfrac{x}{5} + 7$

   The answer to b is correct

9 **a** **i** $\dfrac{x}{4} + 5$ or $\dfrac{1}{4}x + 5$ **ii** $\dfrac{3x}{5} - 2$ or $\dfrac{3}{5}x - 2$

     iii $1 + \dfrac{x}{2}$ or $1 + \dfrac{1}{2}x$ **iv** $11 - \dfrac{5x}{6}$ or $11 - \dfrac{5}{6}x$

   b **i** half of x subtract 9

     ii two-thirds of x add 10

     iii 25 subtract two-ninths of x

     iv 12 add seven-tenths of x

10 **a** perimeter $= 16w + 2v + 6\,$cm
        area $= 8vw + 24w\,$cm^2

   b perimeter $= 18x + \dfrac{5}{4}y\,$cm

       area $= \dfrac{45}{8}xy\,$cm^2

11 $\dfrac{5}{2}a - \dfrac{3}{2}b$

12 a $p + 3l + 2r$

   b $3p + \dfrac{r}{4}$ or $3p + \dfrac{1}{4}r$

   c $\dfrac{r}{5}$ or $\dfrac{1}{5}r$

   d $\dfrac{3r}{5} + \dfrac{3l}{4}$ or $\dfrac{3}{5}r + \dfrac{3}{4}l$

13 a $8\left(\dfrac{y}{4} + 3\right)$ **b** $4\left(\dfrac{y}{3} + 8\right)$

   c $8\left(\dfrac{3y}{4} + 4\right)$ **d** $4\left(\dfrac{3y}{8} + 3\right)$

Exercise 2.2

1 A and **iii**, B and **vi**, C and **i**, D and **ii**, E and **iv**, F and **v**

2 a 7   **b** 1   **c** 9

3 a 13   **b** 17   **c** 72

   d 8   **e** 20

4 a 10   **b** 2   **c** −9

   d −7   **e** −2   **f** 7

   g 25   **h** −22   **i** −22

   j 30   **k** −5   **l** 12

5 a 27   **b** −16

6 a 10   **b** −6   **c** 25

   d −11   **e** 48   **f** 501

   g 8   **h** 640   **i** 6

   j 100   **k** 38   **l** 10

7 a i number of seconds = 60 × number of minutes

    ii $S = 60M$

  b 1800 seconds

8 $d = 70$

9 a She has added 6 and 12 instead of multiplying.

  b $V = 24$

10 $A = 24$

11 Neither, their volumes are the same. Pyramid A: $V = 32\,\text{cm}^3$, pyramid B: $V = 32\,\text{cm}^3$

12 a B $x = y + 8$ **b** B $x = \dfrac{y}{k}$

  c A $x = y - w$ **d** C $x = ry$

  e C $x = \dfrac{y - t}{2}$

13 $x - 5$ has a value of −9. All the others have a value of 9.

14 a $x = 0, 1$ **b** $x = 4$

   c $x = 0$ **d** $x = 0$

15 a $D = 19$ **b** $p = \dfrac{D - 4}{w}$

   c $p = 8$

16 a $s = 75$ **b** $s = 100$

Exercise 2.3

1 a 4×18

×	10	8
4	40	32

$4 \times 18 = 40 + 32 = 72$

  b 3×21

×	20	1
3	60	3

$3 \times 21 = 60 + 3 = 63$

2 a $6 \times 58 = 6 \times (50 + 8)$

×	50	8
6	300	48

$6 \times 58 = 300 + 48 = 348$

  b $6 \times 58 = 6 \times (60 - 2)$

×	60	−2
6	360	−12

$6 \times 58 = 360 + -12 = 348$

3 a $3(x + 5)$

×	x	5
3	$3x$	15

$3(x + 5) = 3x + 15$

  b $2(x + 9)$

×	x	9
2	$2x$	18

$2(x + 9) = 2x + 18$

c $5(y-1)$

×	y	-1
5	$5y$	-5

$5(y-1) = 5y-5$

d $4(y-8)$

×	y	-8
4	$4y$	-32

$4(y-8) = 4y-32$

4 a $3(2x+1)$

×	$2x$	1
3	$6x$	3

$3(2x+1) = 6x+3$

b $5(4x+9)$

×	$4x$	9
5	$20x$	45

$5(4x+9) = 20x+45$

c $2(3y-7)$

×	$3y$	-7
2	$6y$	-14

$2(3y-7) = 6y-14$

d $5(8y-5)$

×	$8y$	-5
5	$40y$	-25

$5(8y-5) = 40y-25$

5 a $6a+36$ **b** $5b+35$

c $7c-56$ **d** $6d-54$

e $40+5e$ **f** $49+7f$

g $36-6g$ **h** $35-5h$

6 a $56i+63$ **b** $48+42j$

c $30k-35$ **d** $56-63l$

e $54a+48m$ **f** $35b+30n$

g $49c-56x$ **h** $54px+48y$

7 No, $4a-28$ is not the same as $28-4a$

8 a $14a+114$ **b** $38b+92$

c $70c+128$ **d** $48d+7$

e $-20e-33$ **f** $108f+33g$

9 a a^2+a **b** b^2-5b

c $3c^2+6c$ **d** $4e^2+9e$

e $3i^2+7ix$ **f** $3aj-7j^2$

g $3k^2-6kx$ **h** $3m^2+9mx$

i $9r^2-3rx-9r$ **j** $6a+4a^2+2ab$

k $-3xz-3xy-3x^2$

10 Equivalent to $40y+48y^2$ are: **A, C, E, H**

Equivalent to $20y^2+24y^3$ are: **B, D, F, G**

11 a $8x+4\,\text{cm}^2$ **b** $6y^2-4y\,\text{cm}^2$

12 a $2a^2+7a$ **b** $5b^2+8b$

c $8c^2+10c$ **d** $2d^2-d$

e $9e-e^2$ **f** $39fg-27f^2$

13 a Q1. The expansion $3a+15-9a-15$ is correct, but he has not collected like terms correctly.

Q2. The expansion $4pq+pr+2qr-4pq$ is correct, but he has not collected like terms correctly.

Q3. The expansion $5b^2+15ab+4a^2+6ab$ is correct, but he has not collected like terms correctly.

b Q1. $-6a$, Q2. $pr+2qr$, Q3. $4a^2+5b^2+21ab$

14 Area $= 3x(3x+4)+2x(2x-1)$

$= 9x^2+12x+4x^2-2x$

$= 13x^2+10x$

15 a $4(3x+7) = 12x+28$

b $3x(2x-1) = 6x^2-3x$

c $6(5x-3) = 30x-18$

d $5x(9-x) = 45x-5x^2$

e $2(2x+4)+3(4x-8) = 16x-16$

f $x(4x+1)-2x(x-5) = 2x^2+11x$

Exercise 2.4

1 a

×	x	6
2	$2x$	12

$2(x+6) = 2x+12$

b

×	x	5
3	$3x$	15

$3(x+5) = 3x+15$

c

×	y	–3
5	$5y$	–15

$5(y-3) = 5y-15$

d

×	y	–7
4	$4y$	–28

$4(y-7) = 4y-28$

2 a $2x+12 = 2(x+6)$

b $3x+15 = 3(x+5)$

c $5y-15 = 5(y-3)$

d $4y-28 = 4(y-7)$

3 a $2x+8 = 2(x+4)$ **b** $3x+9 = 3(x+3)$

c $5y-25 = 5(y-5)$ **d** $7y-14 = 7(y-2)$

4 a $3(2x+1) = 6x+3$

b $4(3x+1) = 12x+4$

c $2(5y-1) = 10y-2$

d $6(4y-1) = 24y-6$

5 a $6x+3 = 3(2x+1)$

b $12x+4 = 4(3x+1)$

c $10y-2 = 2(5y-1)$

d $24y-6 = 6(4y-1)$

6 a $4x+6 = 2(2x+3)$

b $6x-15 = 3(2x-5)$

c $35y+10 = 5(7y+2)$

d $28y-63 = 7(4y-9)$

7 a $5(z+3)$ **b** $2(y-7)$

c $4(5x+1)$ **d** $3(3w-1)$

e $2(3v+4)$ **f** $7(2a-3)$

g $6(2-b)$ **h** $7(2+3d)$

8 A and **iii**, B and **iv**, C and **ii**, D and **i**

9 a $m(7m+1)$ **b** $5a(a-3)$

c $t(t+9)$ **d** $4h(2-h)$

e $3y(1+4y)$ **f** $4y(3-4y)$

g $8e(2e+1)$ **h** $3(5e+2i)$

10 a $14cd-7c = 7c(2d-1)$

b $12a+8ab = 4a(3+2b)$

c $21g+15gh = 3g(7+5h)$

d $30w-15tw = 15w(2-t)$

11 a $2a+4h+8 = 2(a+2h+4)$

b $5b-25+5j = 5(b-5+j)$

c $12tu+16u-20 = 4(3tu+4u-5)$

d $3e^2+4e+ef = e(3e+4+f)$

e $7k-k^2-ak = k(7-k-a)$

f $6n^2-9n+3mn = 3n(2n-3+m)$

12 a Top left: $4x(6+8x)$
Top right: $2(12x+16x^2)$

Bottom left: $x(24+32x)$
Bottom right: $8x(3+4x)$

b Bottom right: $8x(3+4x)$

13 a $7x+7$ **b** $7(x+1)$

14 Correct solution:
$5(3x-2)-5(2+x) = 15x-10-10-5x$
$$= 10x-20$$
$$= 10(x-2)$$

She has made a mistake on the first line of the expansion. Her last term is $+5x$ and it should be $-5x$.

She has done:
$5(3x-2)-5(2+x) = 15x-10-10+5x$
$$= 20x-20$$
$$= 20(x-1)$$

15 $2a(3a+4)-4(a^2+4)+6a(a-8) = 8(a^2-5a-2)$

16 a length $= 2b-5$

b perimeter $= 16b-10$

Exercise 2.5

1 **a** expression **b** formula

   c expression **d** equation

2 **a**

x ⟩×2⟩ $2x$ ⟩+1⟩ 11

5 ⟨÷2⟨   10 ⟨−1⟨ 11

$x = 5$

b

x ⟩×5⟩   ⟩−2⟩ 18

4 ⟨÷5⟨   20 ⟨+2⟨ 18

$x = 4$

c

x ⟩+4⟩   ⟩×3⟩ 21

3 ⟨−4⟨   7 ⟨÷3⟨ 21

$x = 3$

d

x ⟩÷4⟩   ⟩−1⟩ 5

24 ⟨×4⟨   6 ⟨+1⟨ 5

$x = 24$

3

x ⟩×3⟩   ⟩+2⟩ 26

8 ⟨÷3⟨   24 ⟨−2⟨ 26

y ⟩÷2⟩   ⟩+5⟩ 15

20 ⟨×2⟨   10 ⟨−5⟨ 15

$x = 8, \ y = 20$

4

x ⟩+2⟩   ⟩×4⟩ 40

8 ⟨−2⟨ 10 ⟨÷4⟨ 40

y ⟩+3⟩   ⟩×6⟩ 30

2 ⟨−3⟨ 5 ⟨÷6⟨ 30

$x = 8, \ y = 2$

5 **a** $3x + 5 = 17$

x ⟩×3⟩   ⟩+5⟩ 17

$x = 4$ ⟨÷3⟨ 12 ⟨−5⟨ 17

  b $5x + 2 = 27$

x ⟩×5⟩   ⟩+2⟩ 27

$x = 5$ ⟨÷5⟨ 25 ⟨−2⟨ 27

  c $2x − 4 = 12$

x ⟩×2⟩   ⟩−4⟩ 12

$x = 8$ ⟨÷2⟨ 16 ⟨+4⟨ 12

6 **a** $\dfrac{x}{2} + 1 = 20$

$$\frac{x}{2} + 1 - 1 = 20 - 1$$
$$\frac{x}{2} = 19$$
$$x = 19 \times 2$$
$$x = 38$$

  b $\dfrac{x}{3} - 2 = 9$

$$\frac{x}{3} - 2 + 2 = 9 + 2$$
$$\frac{x}{3} = 11$$
$$x = 11 \times 3$$
$$x = 33$$

c $\dfrac{x}{4} - 8 = 16$

$\dfrac{x}{4} - 8 + 8 = 16 + 8$

$\dfrac{x}{4} = 24$

$x = 24 \times 4$

$x = 96$

7 **a** $a = 8\,\text{cm}$ **b** $b = 50\,\text{cm}$

  c $c = 6\,\text{cm}$ **d** $d = 8\,\text{cm}$

8 **a** $x = 5\,\text{cm}$ **b** $x = 4\,\text{cm}$

  c $x = 3\,\text{cm}$

9 **a** $c = 2\,\text{cm},\ d = 50\,\text{cm}$

  b $e = 7\,\text{cm},\ f = 50\,\text{cm}$

  c $i = 5\,\text{cm},\ j = 4\,\text{cm}$

10 **a** $\dfrac{x}{2} - 9 = 5,\ x = 28$

  b $4x - 1 = 3x + 6,\ x = 7$

  c $8(x - 2) = 16(x - 5),\ x = 8$

11 **a** $4(2y + 7) = 52$ or $8y + 28 = 52$

  b $y = 3$

  c $4(2y + 7) = 4(2 \times 3 + 7) = 52$

12 $y = 104$

13 **a** $x = 14$

  b **i** $x = -30$ **ii** $x = 5$

14 **a** $y = 40$ **b** $z = 14$

  c $n = 2$ **d** $m = 12$

15

B	O	B	S	L	E	I	G	H
8	11	8	3	7	4	5	2	9

Exercise 2.6

1 **a** True   **b** False

  c True   **d** False

2 **A** and **iii**, **B** and **i**, **C** and **iv**, **D** and **ii**

3 **a** $8 \leqslant x < 12$ **b** $1 < y < 7$

  c $0 \leqslant m \leqslant 5$ **d** $0 < n \leqslant 5$

4 **a** x is greater than 7 and less than or equal to 15

  b y is greater than 10 and less than 20

c x is greater than or equal to 0 and less than or equal to 5

d y is greater than or equal to 50 and less than 100

5 **A** and **iii**, **B** and **iv**, **C** and **ii**, **D** and **i**

6 **a**

  b

  c

  d
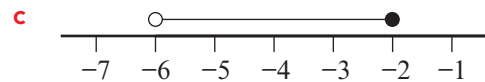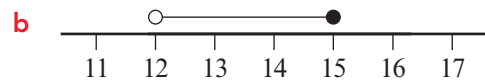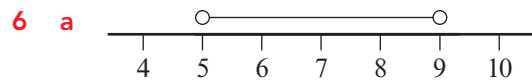
7 **a** $25 \leqslant x \leqslant 28$ **b** $30 < x < 34$

  c $-15 < x \leqslant -10$ **d** $-3 \leqslant x < 1$

8 **a** $x > 4$ is equivalent to $2x > 8$

  b $x < 9$ is equivalent to $7x < 63$

  c $y \geqslant 1$ is equivalent to $y + 9 \geqslant 10$

  d $y \leqslant 1$ is equivalent to $y - 5 \leqslant -4$

9 **i**   smallest integer is -2 and not -3

  ii   largest integer is 2 not 3

  iii x could be $-2, -1, 0, 1, 2$

10 **a** **i**   33   **ii**   37

  iii   33, 34, 35, 36, 37

  b **i**   25   **ii**   27

  iii   25, 26, 27

  c **i**   40   **ii**   43

  iii   40, 41, 42, 43

  d **i** -12 **ii** -9

  iii $-12, -11, -10, -9$

11 **a** T   **b** T   **c** F   **d** F

12 **a** **i**   smallest integer is 6 not 5

  ii   largest integer is 8 not 9

  iii n could be 6, 7, 8

  b **A i** 7 **ii** 10 **iii** 7, 8, 9, 10

  B i -7 **ii** -4 **iii** $-7, -6, -5, -4$

13 answers are in rows

Inequality	Smallest integer	Largest integer	List of integers
$1.9 \leqslant x \leqslant 5.5$	2	5	2, 3, 4, 5
$0.2 < x < 6.1$	1	6	1, 2, 3, 4, 5, 6
$-0.5 < x \leqslant 4.9$	0	4	0, 1, 2, 3, 4
$2.95 \leqslant x < 7.85$	3	7	3, 4, 5, 6, 7

14 a

b
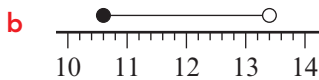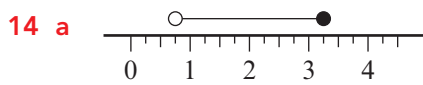
15 a $22.5 \leqslant y \leqslant 25.75$ **b** $0.75 < y < 3.25$

16 a i $12 < y < 18$ **ii** $18 > y > 12$

   b i $0 \leqslant y \leqslant 4$ **ii** $4 \geqslant y \geqslant 0$

   c i $7 < x \leqslant 25$ **ii** $25 \geqslant x > 7$

   d i $10 \leqslant x < 38$ **ii** $38 > x \geqslant 10$

Exercise 3.1

1 a 2   **b** 7   **c** 8   **d** 7.5

2 a 3   **b** 5   **c** 8   **d** 6.5

3 a 12   **b** 1.2   **c** 120   **d** 0.12

4 a 40   **b** 70   **c** 200   **d** 250

5 a 200   **b** 500   **c** 3000   **d** 1200

6 a 160   **b** 1.6   **c** 16   **d** 1600

7 a 3.3   **b** 99.9   **c** 3

   d 0.87   **e** 0.77   **f** 0.7

   g 7   **h** 0.07

8 a 50   **b** 56   **c** 556

   d 5.5   **e** 500   **f** 560

   g 5560   **h** 55

9 a 2.7   **b** 0.279   **c** 2   **d** 270

10 a ÷   **b** ×   **c** ÷   **d** ×

   e ÷   **f** ×

11 a 0.1   **b** 0.1   **c** 0.01   **d** 0.1

   e 0.01   **f** 0.01

12 D is the odd one out as it equals 9600. **A**, **B** and **C** all equal 0.96

13 a $0.12 \, \text{m}^2$ **b** $1.95

14 a $b = \dfrac{2A}{h}$ **b** $23.2 \, \text{m}$

15 2.34

16 a $0.1 \div 0.1 = 1$ which is not bigger than 1

   b learners' examples; any number smaller than 1.00

Exercise 3.2

1 a 200   **b** 5000

   c 20 000   **d** 200 000

2 a 210   **b** 4800

   c 24 000   **d** 190 000

3 a 4730   **b** 66 500   **c** 2 360 000

4 a 0.02   **b** 0.006

   c 0.000 04   **d** 0.7

5 a 0.023   **b** 0.0057

   c 0.000 038   **d** 0.69

6 a **C** 500   **b** **B** 9

   c **A** 6000   **d** **C** 0.004

7 a 360   **b** 0.36   **c** 3600

   d 0.0036   **e** 36   **f** 3.6

8 a Part a: he has forgotten to add the extra zeros. Part b: he has rounded to 3 d.p. not 3 s.f.

   b Part a: 2 370 000   Part b: 0.002 06

9 a 2000   **b** 760   **c** 5.37

   d 0.08   **e** 0.20   **f** 6.04

   g 1000   **h** 0.90   **i** 20.0

10 a **D** 600   **b** **A** 15   **c** **C** 0.0789

   d **D** 0.007 778   **e** **A** 0.040

11 a 762.204 903

   b i 800   **ii** 760   **iii** 762

   iv 762.2   **v** 762.20   **vi** 762.205

12 2700 km

13 a 500 **b** 530 **c** 530

d 530.4 **e** 530.40 **f** 530.404

14 0.0259 g

15 200 000

16 1 100 000 (2 s.f.)

17 0.053 (2 s.f.)

18 a i 120 **ii** 119

b i 400 **ii** 401

c i 12 000 **ii** 12 600

d i 80 **ii** 83.6

e i 1000 **ii** 962

f i 3 **ii** 2.89

g i 25 **ii** 18.6

h i 4 **ii** 5.19

i i 20 **ii** 17.2

Exercise 4.1

1 51, 08, 21, 17

08, 17, 21, 51

5.08, 5.17, 5.21, 5.51

2 a 29 16 95 91

16 29 91 95

4.16 4.29 4.91 4.95

b 94 49 95 47

47 49 94 95

8.47 8.49 8.94 8.95

c 19 15 13 01

01 13 15 19

0.01 0.13 0.15 0.19

3 a 7.27 > 7.23 **b** 9.71 < 9.83

c 20.17 > 20.09 **d** 3.9 > 3.65

4 a −5.2 > −5.7 **b** −6.5 < −6.2

c −7.2 > −7.5 **d** −8.8 > −8.9

5 a 3.5 g > 380 mg

b 0.4 t < 845 kg

c 2.5 cm < 48 mm

d 950 g > 0.08 kg

e 2500 m > 1.9 km

f 250 cm < 6.5 m

6 a 45.399, 45.454, 45.545, 45.933

b 5.009, 5.044, 5.077, 5.183

c 31.14, 31.148, 31.41, 31.425

d 7.02, 7.052, 7.2, 7.502

7 a 205.5 cm ≠ 255 mm **b** 0.125 g = 125 mg

c 500 g ≠ 0.05 kg **d** 10.5 t ≠ 1050 kg

e 0.22 kg = 220 g **f** 1.75 km ≠ 175 m

8 a 9.1 > 9.03

b 56.4 > 56.35

c 0.66 > 0.606

d 3.505 < 3.7

e 0.77 t < 806 kg

f 7800 m > 0.8 km

g 3.5 kg > 375 g

h 156.3 cm > 1234 mm

9 a 0.2 cm, 7 mm, 27 mm, 4.3 cm

b 19.5 mm, 29 cm, 34.5 cm, 500 mm

c 2000 g, 3 kg, 5550 g, 75.75 kg

d 0.9 kg, 1.75 kg, 1800 g, 1975 g

e 100 mg, 0.125 g, 150 mg, 0.2 g

f 0.05 km, 999 m, 2750 m, 25 km

10 a −2.3 > −2.4 **b** −7.23 > −7.29

c −0.15 < −0.08 **d** −11.02 > −11.5

11 a −8.8, −8.34, −8.28, −8.06

b −1.78, −1.5, −1.425, −1.03

12 a 32 km as it is a lot more than the other numbers.

b Sarina is wrong. Longest distance = 4 km, shortest distance = 0.5 km

0.5 × 10 = 5 km which is > 4 km, not < 4 km

c   Sarina runs in the 250 m park as her distances are all multiples of 250 m.

Frank runs in the 400 m park as his distances are all multiples of 400 m.

13 a   A −6.9, **B** −6.84, **C** −6.95

b   C −6.95, A −6.9, B −6.84

14   −1.43, −1.42, −1.41, −1.40, −1.39

15 a $F = -37.48$ when $C = -38.6$

b   −38.6 °F is colder, because −38.6 °C = −37.48 °F which is warmer than −38.6 °F

Exercise 4.2

1 a $\times 0.4$ is the same as $\div 10$ and $\times 4$ OR $\times 4$ and $\div 10$

b $\times 0.6$ is the same as $\div 10$ and $\times 6$ OR $\times 6$ and $\div 10$

2 a $30 \div 10 = 3$ and $3 \times 2 = 6$

b $-40 \div 10 = -4$ and $-4 \times 2 = -8$

c $12 \times 2 = 24$ and $24 \div 10 = 2.4$

d $-8 \times 2 = -16$ and $-16 \div 10 = -1.6$

3 a $30 \div 10 = 3$ and $3 \times 3 = 9$

b $-50 \div 10 = -5$ and $-5 \times 3 = -15$

c $15 \times 3 = 45$ and $45 \div 10 = 4.5$

d $-9 \times 3 = -27$ and $-27 \div 10 = -2.7$

4 a $500 \div 100 = 5$ and $5 \times 2 = 10$

b $-600 \div 100 = -6$ and $-6 \times 2 = -12$

c $25 \times 2 = 50$ and $50 \div 100 = 0.5$

d $-4 \times 2 = -8$ and $-8 \div 100 = -0.08$

5 a $500 \div 100 = 5$ and $5 \times 3 = 15$

b $-700 \div 100 = -7$ and $-7 \times 3 = -21$

c $12 \times 3 = 36$ and $36 \div 100 = 0.36$

d $-3 \times 3 = -9$ and $-9 \div 100 = -0.09$

6 a   −0.9   **b**   1.5   **c**   −6   **d**   4.2

e   −7.2

7 a   −0.24   **b**   −2.4

c   −0.024   **d**   −24

8   **E** −13.5, **D** −13, **C** −12.9, **B** −12.6, **A** −12.48

9 a i $3 \times 3 = 9$

$0.3 \times 3 = 0.9$

$0.3 \times 0.3 = 0.09$

$0.3 \times 0.03 = 0.009$

$0.3 \times 0.003 = 0.0009$

ii $4 \times 7 = 28$

$0.4 \times 7 = 2.8$

$0.4 \times 0.7 = 0.28$

$0.4 \times 0.07 = 0.028$

$0.4 \times 0.007 = 0.0028$

b i   0.005   **ii**   0.24   **iii**   0.024

iv   0.0016   **v**   0.048   **vi**   0.006

10 a   Equal to 0.0012 are: **A, G, H**

Equal to 0.016 are: **B, E, I**

Equal to 0.0018 are: **C, D, J**

F is left over. $0.05 \times 0.4 = 0.02$

b   Learners' answers. Any calculation that gives an answer of 0.02, e.g. 0.2×0.1

11 a   13 104

b i   1310.4   **ii**   1310.4

iii   131.04   **iv**   13.104

v   13.104   **vi**   0.131 04

12 a   Estimate: $7 \times 8 = 56$, Accurate: 59.76

b   Estimate: $0.2 \times 5 = 1$, Accurate: 1.08

c   Estimate: $0.9 \times 6 = 5.4$, Accurate: 5.5245

d   Estimate: $0.6 \times 0.7 = 0.42$, Accurate: 0.452 62

13 a   Estimate $= 0.2 \times 7 = 1.4$, so 1.587 could be correct

b   Estimate: $80 \times 0.003 = 0.24$ so 0.0246 is incorrect

c   Estimate: $0.08 \times 0.005 = 0.0004$, so 0.0039 is incorrect

14 a   Estimate: $30 \times 2 = 60$ ml

b   75 ml

15 a   Estimate: $33 \times 0.03 = 0.99$ litres

b   0.975 litres

16 a i $F = 5$ when $C = -15$

ii $F = -4$ when $C = -20$

b Marcus is not correct. When $C = -17$, $F = 1.4$. The closest to zero is when $C = -18$ and $F = -0.4$ because -0.4 is closer to zero than 1.4

Exercise 4.3

1 a $1.6 \div 0.4 = \dfrac{1.6}{0.4}, \dfrac{1.6 \times 10}{0.4 \times 10} = \dfrac{16}{4} = 4$

b $4.5 \div 0.9 = \dfrac{4.5}{0.9}, \dfrac{4.5 \times 10}{0.9 \times 10} = \dfrac{45}{9} = 5$

c $-24 \div 0.3 = \dfrac{-24}{0.3}, \dfrac{-24 \times 10}{0.3 \times 10} = \dfrac{-240}{3} = -80$

d $-21 \div 0.7 = \dfrac{-21}{0.7}, \dfrac{-21 \times 10}{0.7 \times 10} = \dfrac{-210}{7} = -30$

2 **A** and **iii**, **B** and **i**, **C** and **v**, **D** and **ii**, **E** and **iv**

3 a $2 \div 0.4 = \dfrac{2}{0.4}, \dfrac{2 \times 10}{0.4 \times 10} = \dfrac{20}{4} = 5$

b $3 \div 0.5 = \dfrac{3}{0.5}, \dfrac{3 \times 10}{0.5 \times 10} = \dfrac{30}{5} = 6$

c $-6 \div 0.2 = \dfrac{-6}{0.2}, \dfrac{-6 \times 10}{0.2 \times 10} = \dfrac{-60}{2} = -30$

d $-4 \div 0.8 = \dfrac{-4}{0.8}, \dfrac{-4 \times 10}{0.8 \times 10} = \dfrac{-40}{8} = -5$

4 a She has not multiplied the 40 by 10

b 80

5 **C** is the odd one out as the answer is 110. All the others have an answer of 120.

6 a 2.6 **b** 16.4 **c** −1230 **d** −270

7 $4.30

8 a i Estimate: $51 \div 0.3 = 170$

ii Accurate: 165

b i Estimate: $-900 \div 0.4 = -2250$

ii Accurate: −2340

c i Estimate: $30 \div 0.5 = 60$

ii Accurate: 63

d i Estimate: $-360 \div 0.6 = -600$

ii Accurate: −585

e i Estimate: $56 \div 0.7 = 80$

ii Accurate: 84.2

f i Estimate: $-4000 \div 0.8 = -5000$

ii Accurate: −4760

9 a

1	2	3	4	5	6	7	8	9
13	26	39	52	65	78	91	104	117

b 58.1 **c** $60 \times 13 = 780$

10 a

1	2	3	4	5	6	7	8	9
19	38	57	76	95	114	133	152	171

b $24.80

c $25 \times 2 = 50

11 a i 654 **ii** 32

iii 6540 **iv** 320

b learners' answers

c i 654 **ii** 65.4

iii 6.54 **iv** 0.654

d learners' answers

12 a 4.2 (1 d.p.) **b** 59.18 (2 d.p.)

c −3043.889 (3 d.p.)

13 a learners' own proof, e.g. $0.5 \times 5.2 \times 3.64 = 2.6 \times 3.64 = 9.464\,\text{m}^2$ and $9.464\,\text{m}^2 \neq 8.84\,\text{m}^2$

b height $= 3.4\,\text{m}$

14 2.4 m

15 a No. Learners' explanations, e.g. $7.2 \times 0.8 = 5.76$ and $5.76 \neq 8.64$

b term-to-term rule is: multiply by 1.2, 1st term $= 6$, 4th term $= 10.368$; learners' explanations

Exercise 4.4

1 a $(0.2 + 0.1) \times 0.4$

$= 0.3 \times 0.4$

$= 0.12$

b $(0.9 - 0.7) \times 0.3$

$= 0.2 \times 0.3$

$= 0.06$

c $(0.4 + 0.5) \times 0.6$

$= 0.9 \times 0.6$

$= 0.54$

d $(0.8 - 0.6) \times 0.8$

$= 0.2 \times 0.8$

$= 0.16$

2 a 60×0.9

$= 60 \times (1 - 0.1)$

$= 60 \times 1 - 60 \times 0.1$

$= 60 - 6$

$= 54$

b 42×0.9

$= 42 \times (1 - 0.1)$

$= 42 \times 1 - 42 \times 0.1$

$= 42 - 4.2$

$= 37.8$

c 18×0.9

$= 18 \times (1 - 0.1)$

$= 18 \times 1 - 18 \times 0.1$

$= 18 - 1.8$

$= 16.2$

d 24×0.9

$= 24 \times (1 - 0.1)$

$= 24 \times 1 - 24 \times 0.1$

$= 24 - 2.4$

$= 21.6$

3 **a** 57 **b** 241 **c** 24 **d** 3.9

e 48 **f** 35 **g** 24 **h** 12

4 **a** 16×0.35

$= 16 \times 0.5 \times 0.7$

$= 8 \times 0.7$

$= 8 \times 7 \div 10$

$= 56 \div 10$

$= 5.6$

b 12×0.45

$= 12 \times 0.5 \times 0.9$

$= 6 \times 0.9$

$= 6 \times 9 \div 10$

$= 54 \div 10$

$= 5.4$

c 18×0.15

$= 18 \times 0.5 \times 0.3$

$= 9 \times 0.3$

$= 9 \times 3 \div 10$

$= 27 \div 10$

$= 2.7$

d 26×0.35

$= 26 \times 0.5 \times 0.7$

$= 13 \times 0.7$

$= 13 \times 7 \div 10$

$= 91 \div 10$

$= 9.1$

5 **a** 0.24 **b** 0.56

c 0.66 **d** 0.48

6 **a** 14.4 **b** 32.4 **c** 4.68

7 $7.83\,\text{m}^2$

8 **a** 475.2 **b** 47.52

c 11.88 **d** 1.188

9 3.96 m

10 29.7 m

11 **a** 265 **b** 1268

c 6.4 **d** 25.26

e 135 **f** 12

12 **a** 10.5 **b** 43.4

13 **a** $S = 14$ **b** $a = 7.2$

14 **a** 39.6 m **b** $98.01\,\text{m}^2$

15 **a** $F = 11.7$ **b** $m = 52$

16 **a** 56 **b** 27.3

17 $a = 425,\ b = 51,\ c = 840,\ d = 378,\ e = 26.25$

Exercise 5.1

1 $a + b = 180°$, angles on a straight line, so $a = 180 - b$; $b + c = 180°$, angles on a straight line, so $c = 180 - b$; hence a and c are equal.
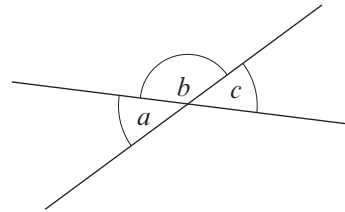
2 **a** They are vertically opposite angles

b

c
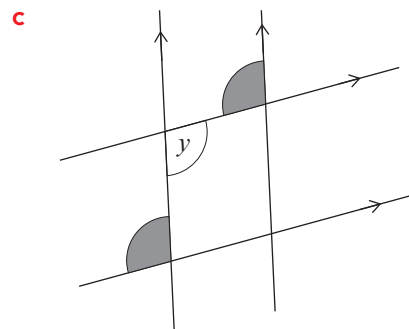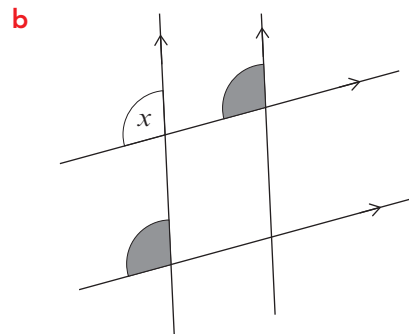
3 $a = 75°$, vertically opposite angles; $b = 75°$, corresponding to the given angle; $c = 105°$, angles on a straight line; $d = 105°$, alternate angle to c

4 **a** g and i **b** c and e

5 **a** **i** *BEF* **ii** *DEB*

iii *EBC*

b No. Alternate angles must be between the parallel lines AC and DF. Sofia's angles are not.

6 Lines l and n are parallel because corresponding angles (80° and 100°) are equal. These angles are not the same for line m so that is not parallel to the other two.

7 $a = 118 - 74 = 44°$, exterior angle;

so $x = 44°$, vertically opposite angle
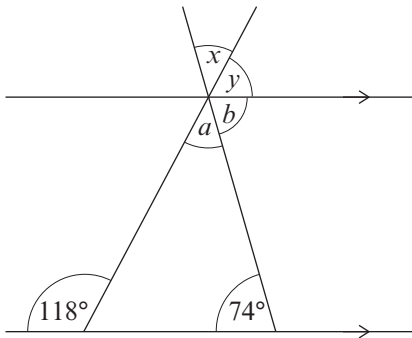
$b = 74°$, alternate angle;

$y = 180 - 44 - 74 = 62°$, angles on a straight line

8 $s = 120°$, vertically opposite angles;
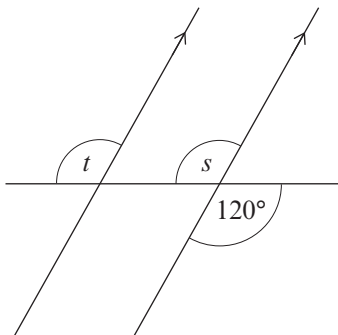
$s = t$, corresponding angles;

hence $t = 120°$; other explanations are possible

9 Yes. The angle vertically opposite 50° is also 50°. That angle and the 75° add up to 125° and that makes an angle alternate to the 125° marked. Other explanations are possible.

10 $a = c$, corresponding angles;
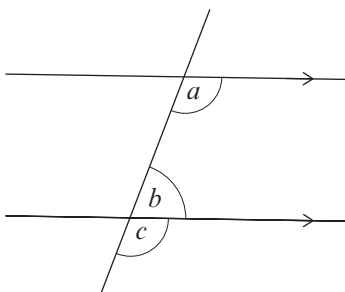
$b + c = 180°$, angles on a straight line;

hence $a + b = 180°$

11 Angle $WXV =$ angle Y, corresponding angles; angle $VXZ =$ angle Z, alternate angles; $WXV + VXZ + ZXY = 180°$, angles on a straight line; hence $X + Y + Z = 180°$.

12 alternate angles; alternate angles; angles on a straight line

13 a $x = y$, alternate angles;
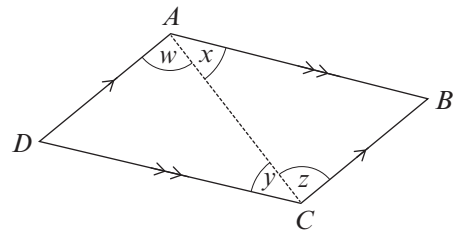
$w = z$, alternate angles;

hence $x + w = y + z$ or $A = C$

b Draw diagonal BD and prove $B = D$ using similar arguments to those in part a.

Exercise 5.2

Example explanations are given for some of the proofs in this exercise. There may be other acceptable proofs.

1 120°

2 Each one is 165°

3 $x = 75°$ and $y = 40°$

4 $a = 110°$ and $b = 70°$

5 Divide the shape into two triangles. Show the angles of the two triangles are the same as the four angles of the quadrilateral.

6 $a = c + d$, exterior angle; $e = g + h$, exterior angle; $a + e + f + b = 360°$, angles at a point; hence $c + d + g + h + f + b = 360°$ and these are the angles of $PQRS$.

7 a The six angles of the two triangles add up to $2 \times 180 = 360°$. The four angles A, B, C and D must be less than this.

b The 120° angle is the exterior angle of both triangles. A and D add up to 120° and so do B and C. Hence $A + B + C + D = 240°$.

8 $x = a + c$, exterior angle;
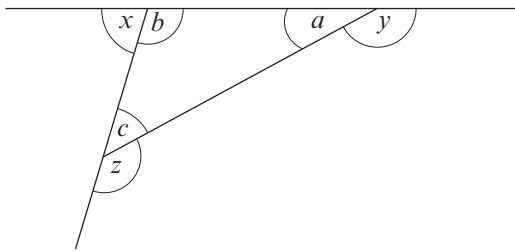
$y = b + c$, exterior angle;

$z = b + a$, exterior angle;

$x + y + z = a + c + b + c + b + a =$
$2(a + b + c) = 2 \times 180 = 360°$

9 $a = 110 - 40 = 70°$, exterior angle of a triangle;
$b = 110 - 55 = 55°$, exterior angle;
$c = 110 - 70 = 40°$, exterior angle

10 a $115°$

b
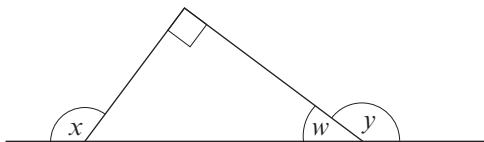
External angle $x = 90° + w$ so $w = x - 90°$

$w + y = 180°$ because they are angles on a straight line, so $x - 90° + y = 180°$

Hence $x + y = 270°$

11 a $75°$, $65°$ and $40°$

b Either: $100° + 120° + 130° = 350°$ and this should be $360°$

or the angle sum is $80° + 60° + 50° = 190°$ and this should be $180°$

Exercise 5.3

For the constructions, accept answers for lengths within 0.2 cm and answers for angles within 2°, to allow for slight drawing errors.

1 a construction **b** 7.4 cm

2 a construction **b** 7.0 cm

3 a construction **b** 49°

c 9.2 cm

4 a construction **b** 13.0 cm

5 a construction **b** 97°

6 a i construction **ii** 10.8 cm

b i construction **ii** 6.7 cm

7 a construction

b $BD = 19.0 \pm 0.1$ cm

8 a construction

b angle opposite 10.5 should measure 90°

9 a, **b** and **c** construction and check

10 a and **b** construction

c 4.4 cm

11 a and **b** construction **c** 8.9 cm

d construction **e** 10.6 cm

Exercise 6.1

Some questions have several possible answers. Examples are given.

1 a gender, date of birth, address

b age in years, test marks

c height, mass

2 a answers will probably include the amount of homework done on different nights, the subjects and the age or year of the learner

b answers should include a range of ages and both boys and girls if you are in a mixed school

3 a gender is categorical, length and mass are continuous

b questions could be about gender differences or about the relationship between length and mass

c put names in a hat or take all the babies born on a particular day or in a particular week

4 a categorical data: reasons for choice and why they are staying

discrete data: the number of nights' stay

b For the categorical data use a questionnaire. This should have categories to tick rather than open-ended questions. The numerical data can come from the questionnaire or from hotel records.

5 Possible answers:

   a How many portions of vegetables a child eats in a day. The age and gender of the child.

   b Interview or give a questionnaire to a sample of children. Equal numbers of boys and girls. Children of different ages. If there are boys and girls in your school you could collect data from them.

6 a You need the time taken to complete a puzzle for boys and girls of the two different ages.

   b Give the same puzzle to a sample of children. If the puzzle is not the same for everyone the times cannot be compared. Record the time taken to complete the puzzle. The sample should include equal numbers of boys and girls and equal numbers of children of both ages.

7 Possible answers:

   a Common words in one language and their translations in at least one other language; or some words and their translations chosen at random from a dictionary.

   b, c and **d** learners' own answers

8 a The number of words in a sample of sentences from a book for children and the same for a book for adults. You could choose more than one book for each.

   b, **c** and **d** learners' own answers

9 Possible factors that could make a difference are age, gender, ethnicity. The sample chosen should take account of these factors so that you can see if the results are different for different groups. You need to have a large enough sample to be confident about your answers.

Exercise 6.2

Some questions have several possible answers. Examples are given.

1 a 1. The first 40 names on a list of students.
2. Take 40 names out of a hat.
3. All the students (up to 40) in one or two lectures.

   b 1 is quick but if you do it again you will get the same people. 2 will give a representative sample but it will take

a long time to do. 3 is quick but will only choose students studying particular subjects.

2 a It is quick and easy to do.

   b The sample will not represent the range of people who use the bus during the day. It only covers one day and that might not be typical.

   c Ask small groups of people at different times and on different days.

   d It will take a lot more of Zara's time and so it will be more expensive.

3 a Yes. Learners' reasons, e.g.

   • The morning has a mode of 1 to 1.5 hours and the afternoon has a mode of 0.5 to 1 hours.

   • In the morning, 65% of the cars park for more than 1 hour; in the afternoon, only 35% of the cars park for more than 1 hour.

   b Learners' suggestions, e.g. sample on other days of the week, in case Tuesdays are not typical.

4 a People in the coffee shop at that time may not be typical.

   b Choose people at different times during the day. In this way you can get a range of people: people going to work, people going shopping during the day, people on their lunch break, people going home in the early evening, etc.

5 a Because the amount of traffic will be different at different times and this will affect speed.

   b The two time periods are both during rush hour so they do not represent the whole day.

   c Survey the cars at more time periods, throughout the day. Survey more than one day.

6 a No. Learners' reasons, e.g.

   • The shapes of the two distributions are approximately the same.

   • Hotel A has 45 out of 80 scores that are 4 or 5; hotel B has 19 out of 36. The fraction is just over half in both cases.

b Learners' suggestions, e.g. Use a bigger sample size for hotel B or any other suitable comment about sample sizes.

7 a You need to get details of each person replying so you can make sure you have a representative sample.

b It is easy to do: you just need a phone and you do not need to travel.

c A lot of people do not answer calls from unknown numbers, or may be unwilling to take part in the survey, so it could take a long time to get enough representative replies. This will make the survey expensive.

8 learners' activity

Exercise 7.1

1 $\frac{1}{2} = 0.5$ terminating, $\frac{1}{3} = 0.\dot{3}$ recurring,

$\frac{1}{4} = 0.25$ terminating, $\frac{1}{5} = 0.2$ terminating,

$\frac{1}{6} = 0.1\dot{6}$ recurring, $\frac{1}{7} = 0.\dot{1}4285\dot{7}$ recurring,

$\frac{1}{8} = 0.125$ terminating, $\frac{1}{9} = 0.\dot{1}$ recurring,

$\frac{1}{10} = 0.1$ terminating

2 a $\frac{2}{5} = 0.4$ terminating

b $\frac{2}{3} = 0.\dot{6}$ recurring

c $\frac{3}{4} = 0.75$ terminating

d $\frac{3}{5} = 0.6$ terminating

e $\frac{5}{6} = 0.8\dot{3}$ recurring

f $\frac{2}{7} = 0.\dot{2}8571\dot{4}$ recurring

g $\frac{3}{8} = 0.375$ terminating

h $\frac{4}{9} = 0.\dot{4}$ recurring

i $\frac{7}{10} = 0.7$ terminating

j $\frac{2}{11} = 0.\dot{1}\dot{8}$ recurring

3 $\frac{2}{11}, \frac{3}{8}, \frac{4}{9}, \frac{3}{5}, \frac{7}{10}$

4 a terminating, with learners' explanations

b $\frac{7}{8} = 0.875, \frac{4}{5} = 0.8, \frac{3}{10} = 0.3, \frac{3}{20} = 0.15,$

$\frac{8}{25} = 0.32$

c $\frac{3}{20}, \frac{3}{10}, \frac{8}{25}, \frac{4}{5}, \frac{7}{8}$

5 a recurring, with learners' explanations

b $\frac{5}{9} = 0.\dot{5}, \frac{1}{3} = 0.\dot{3}, \frac{5}{12} = 0.41\dot{6}, \frac{4}{11} = 0.\dot{3}\dot{6},$

$\frac{8}{15} = 0.5\dot{3}$

c $\frac{1}{3}, \frac{4}{11}, \frac{5}{12}, \frac{8}{15}, \frac{5}{9}$

6 Marcus is incorrect.

learners' explanations, e.g. $\frac{3}{6} = \frac{1}{2} = 0.5$

which is a terminating decimal

7 a $0.\dot{8}$ **b** 0.85

c $0.2\dot{6}$ **d** 0.675

8 a $0.\dot{8}5714\dot{2}$ **b** $0.\dot{8}4615\dot{3}$

c $0.\dot{2}3809\dot{5}$

9 **i** is correct

ii is incorrect: there should be a dot over the 7 as well as the 2, i.e. $0.\dot{7}\dot{2}$

iii is incorrect: she has written the numbers in the wrong order; the correct answer is $0.6\dot{1}$

iv is incorrect: the second dot should be over the 5, not the 1, i.e. $0.\dot{1}2820\dot{5}$

10 learners' explanations, e.g. She is wrong. It is a recurring decimal but the calculator has rounded up the final digit on the screen. $\frac{7}{9} = 0.\dot{7}$

11 $\frac{5}{27} = 0.\dot{1}8\dot{5}$

12 $0.5, \frac{7}{13}, 55\%, 0.56, \frac{4}{7}, 58.2\%, 0.6, \frac{18}{27}$

13 a $1.\dot{6}$ **b** 3.25

c $3.\dot{2}$ **d** 4.375

14 a i $3\frac{1}{2}$ hours   **ii** 3.5 hours

b **i** $2\frac{3}{4}$ hours   **ii** 2.75 hours

c **i** $1\frac{1}{6}$ hours   **ii** $1.1\dot{6}$ hours

d **i** $4\frac{1}{3}$ hours   **ii** $4.\dot{3}$ hours

e **i** $9\frac{1}{5}$ hours   **ii** 9.2 hours

f **i** $11\frac{5}{12}$ hours   **ii** $11.41\dot{6}$ hours

15 learners' explanations, e.g. Arun is wrong and his teacher is correct. The 6 in 0.006 has a recurring dot, so it is 0.0066666 . . . not 0.006 which is what Arun has used. When you double 0.006 you get 0.012, but when you double 0.0066666 . . . you do get 0.013333 . . .

Exercise 7.2

1 **a** $\frac{13}{4} = 3\frac{2}{8}$ **b** $\frac{40}{9} \neq 4\frac{1}{3}$

  c $-\frac{9}{6} = -1\frac{1}{2}$ **d** $-4\frac{3}{5} \neq -\frac{47}{10}$

2 **a** $\frac{7}{2} < 3\frac{3}{4}$ **b** $\frac{13}{3} > 4\frac{1}{6}$

  c $8\frac{2}{5} > \frac{83}{10}$ **d** $\frac{22}{3} < 7\frac{2}{5}$

3 **a** $-\frac{5}{4} > -1\frac{1}{2}$ **b** $-\frac{8}{3} > -2\frac{5}{6}$

  c $-\frac{27}{5} < -5\frac{4}{15}$ **d** $-\frac{17}{6} < -2\frac{3}{4}$

4 **a** $-2\frac{5}{8}$ **b** $-\frac{23}{10}$

  c $-\frac{23}{3}$

5 **a** **i** $-\frac{17}{7} = -2.4285\ldots$

    ii $-\frac{22}{9} = -2.4444\ldots$

    iii $-\frac{27}{11} = -2.4545\ldots$

  b $-\frac{27}{11}, -\frac{22}{9}, -\frac{17}{7}$

6 **a** When you compare two fractions with the same denominator, the larger the numerator the larger the fraction.

  b When you compare two fractions with the same numerator, the larger the denominator the smaller the fraction.

7 **a** $\frac{2}{9} < \frac{7}{9}$ **b** $\frac{15}{8} < \frac{19}{8}$

  c $\frac{7}{11} > \frac{7}{13}$ **d** $\frac{4}{5} < \frac{4}{3}$

8 $-\frac{14}{3}, -\frac{22}{5}, -4\frac{1}{3}, -4\frac{1}{5}$

9 Ben, 74%

10 Dolphins club, 70%

11 $-\frac{107}{12}, -8\frac{7}{8}, -\frac{61}{7}, -\frac{49}{6}$

12 On the second day, 88%

13 Drug A

14 **a** Yes. $-\frac{8}{9} = -\frac{32}{36}$ and $-\frac{13}{18} = -\frac{26}{36}$. Halfway between 26 and 32 is 29, so $-\frac{29}{36}$ is exactly halfway between $-\frac{32}{36}$ and $-\frac{23}{36}$.

  b $-1\frac{19}{24}$

Exercise 7.3

1 **a** $4\frac{2}{3} - 3\frac{1}{3}$ ① $\frac{14}{3} - \frac{10}{3}$ ② $\frac{14}{3} - \frac{10}{3} = \frac{4}{3}$

    ③ $\frac{4}{3} = 1\frac{1}{3}$

  b $3\frac{2}{9} - 1\frac{7}{9}$ ① $\frac{29}{9} - \frac{16}{9}$ ② $\frac{29}{9} - \frac{16}{9} = \frac{13}{9}$

    ③ $\frac{13}{9} = 1\frac{4}{9}$

  c $7\frac{1}{5} - 5\frac{2}{5}$ ① $\frac{36}{5} - \frac{27}{5}$ ② $\frac{36}{5} - \frac{27}{5} = \frac{9}{5}$

    ③ $\frac{9}{5} = 1\frac{4}{5}$

  d $5\frac{3}{7} - 2\frac{6}{7}$ ① $\frac{38}{7} - \frac{20}{7}$ ② $\frac{38}{7} - \frac{20}{7} = \frac{18}{7}$

    ③ $\frac{18}{7} = 2\frac{4}{7}$

2 **a** $1\frac{2}{5}$ **b** $1\frac{7}{11}$ **c** $1\frac{5}{7}$ **d** $2\frac{8}{9}$

3 **a** $4\frac{1}{2} - 2\frac{3}{4}$ ① $\frac{9}{2} - \frac{11}{4}$ ② $\frac{18}{4} - \frac{11}{4} = \frac{7}{4}$

    ③ $\frac{7}{4} = 1\frac{3}{4}$

b $3\frac{1}{8}-1\frac{1}{4}$ ① $\frac{25}{8}-\frac{5}{4}$ ② $\frac{25}{8}-\frac{10}{8}=\frac{15}{8}$

  ③ $\frac{15}{8}=1\frac{7}{8}$

c $5\frac{3}{5}-2\frac{3}{10}$ ① $\frac{28}{5}-\frac{23}{10}$ ② $\frac{56}{10}-\frac{23}{10}=\frac{33}{10}$

  ③ $\frac{33}{10}=3\frac{3}{10}$

d $6\frac{1}{3}-2\frac{1}{6}$ ① $\frac{19}{3}-\frac{13}{6}$ ② $\frac{38}{6}-\frac{13}{6}=\frac{25}{6}$

  ③ $\frac{25}{6}=4\frac{1}{6}$

4 a $4\frac{11}{12}$ **b** $2\frac{13}{16}$

  c $1\frac{1}{2}$ **d** $2\frac{5}{6}$

5 **A** $2\frac{1}{4}$ **B** $2\frac{1}{3}$ **C** $2\frac{1}{3}$

  A gives the different answer

6 a i $2\,\text{km}$ **ii** $2\frac{1}{8}\,\text{km}$

  b i $20\,\text{km}$ **ii** $19\frac{3}{8}\,\text{km}$

7 $\frac{5}{8}\,\text{m}$

8 $8\frac{1}{4}-3\frac{9}{10}$ ① $\frac{33}{4}-\frac{39}{10}$ ② $\frac{33}{4}-\frac{39}{10}=\frac{165}{20}-\frac{78}{20}=\frac{87}{20}$

  ③ $\frac{87}{20}=4\frac{7}{20}$

9 a $3\frac{9}{10}$ **b** $1\frac{7}{18}$

  c $2\frac{11}{12}$ **d** $4\frac{37}{40}$

10 a i $8\,\text{m}$ **ii** $8\frac{3}{20}\,\text{m}$

  b $2\frac{1}{10}\,\text{m}$

11 No, the range is $187\frac{3}{4}-95\frac{7}{10}=92\frac{1}{20}$

12 a She has worked out $3-1=2$ and $\frac{27}{36}-\frac{16}{36}$

  instead of $\frac{16}{36}-\frac{27}{36}$

  b $1\frac{25}{36}$

13 a **B** is taller [**A** is $6\frac{5}{12}$ m, **B** is $6\frac{23}{24}$ m]

  b $\frac{13}{24}$ m

14 a $6\,\text{m}^2$ **b** $6\frac{2}{21}\,\text{m}^2$

15 Any two counter-examples, e.g. $1\frac{3}{4}+1\frac{1}{2}=3\frac{1}{4}$

  and $1+1+1=3$ and $3\frac{1}{4}>3$

  e.g. $2\frac{8}{9}+5\frac{7}{9}=8\frac{2}{3}$ and $2+5+1=8$ and $8\frac{2}{3}>8$

16 a learners' choices of fractions to add – all possible results in table below:

$+$	$2\frac{17}{36}$	$3\frac{17}{18}$	$4\frac{1}{9}$	$5\frac{11}{24}$
$2\frac{17}{36}$		$6\frac{5}{12}$	$6\frac{7}{12}$	$7\frac{67}{72}$
$3\frac{17}{18}$	$6\frac{5}{12}$		$8\frac{1}{18}$	$9\frac{29}{72}$
$4\frac{1}{9}$	$6\frac{7}{12}$	$8\frac{1}{18}$		$9\frac{41}{72}$
$5\frac{11}{24}$	$7\frac{67}{72}$	$9\frac{29}{72}$	$9\frac{41}{72}$	

  b $5\frac{11}{24}+4\frac{1}{9}=9\frac{41}{72}$. It's the two numbers with the greatest whole number parts.

  c learners' choices of fractions to subtract – all possible results in table below:

$-$	$2\frac{17}{36}$	$3\frac{17}{18}$	$4\frac{1}{9}$	$5\frac{11}{24}$
$2\frac{17}{36}$		$1\frac{17}{36}$	$1\frac{23}{36}$	$2\frac{71}{72}$
$3\frac{17}{18}$	$1\frac{17}{36}$		$\frac{1}{6}$	$1\frac{37}{72}$
$4\frac{1}{9}$	$1\frac{23}{36}$	$\frac{1}{6}$		$1\frac{25}{72}$
$5\frac{11}{24}$	$2\frac{71}{72}$	$1\frac{37}{72}$	$1\frac{25}{72}$	

d $4\frac{1}{9} - 3\frac{17}{18} = \frac{1}{6}$. $4\frac{1}{9}$ is just over 4 and $3\frac{17}{18}$ is just under 4 so these two numbers are the closest together. The difference is $\frac{1}{6}$.

Exercise 7.4

1 a $2\frac{1}{2} \times 6 = 2 \times 6 + \frac{1}{2} \times 6$
$= 12 + 3$
$= 15$

b $3\frac{1}{4} \times 8 = 3 \times 8 + \frac{1}{4} \times 8$
$= 24 + 2$
$= 26$

c $5\frac{1}{3} \times 9 = 5 \times 9 + \frac{1}{3} \times 9$
$= 45 + 3$
$= 48$

d $4\frac{1}{5} \times 15 = 4 \times 15 + \frac{1}{5} \times 15$
$= 60 + 3$
$= 63$

2 a $12 \times 2 = 24\,\text{m}^2$

b $27\,\text{m}^2$

3 a $3\frac{2}{3} \times 12 = 3 \times 12 + \frac{2}{3} \times 12$
$= 36 + 8$
$= 44$

b $2\frac{3}{4} \times 8 = 2 \times 8 + \frac{3}{4} \times 8$
$= 16 + 6$
$= 22$

c $3\frac{2}{5} \times 10 = 3 \times 10 + \frac{2}{5} \times 10$
$= 30 + 4$
$= 34$

d $1\frac{5}{6} \times 18 = 1 \times 18 + \frac{5}{6} \times 18$
$= 18 + 15$
$= 33$

4 a Estimate: $20 \times 4 = 80$

b i Archie could be correct as 80 is close to 77

ii He is correct. $21 \times 3\frac{2}{3} = 77$

5 a $3\frac{1}{2} \times 7 = 3 \times 7 + \frac{1}{2} \times 7$
$= 21 + \frac{7}{2}$
$= 21 + 3\frac{1}{2}$
$= 24\frac{1}{2}$

b $4\frac{1}{4} \times 9 = 4 \times 9 + \frac{1}{4} \times 9$
$= 36 + \frac{9}{4}$
$= 36 + 2\frac{1}{4}$
$= 38\frac{1}{4}$

c $6\frac{2}{3} \times 5 = 6 \times 5 + \frac{2}{3} \times 5$
$= 30 + \frac{10}{3}$
$= 30 + 3\frac{1}{3}$
$= 33\frac{1}{3}$

d $4\frac{3}{5} \times 8 = 4 \times 8 + \frac{3}{5} \times 8$
$= 32 + \frac{24}{5}$
$= 32 + 4\frac{4}{5}$
$= 36\frac{4}{5}$

6 Area $= 7 \times 7 + 7 \times 11\frac{3}{4} = 49 + 77 + 5\frac{1}{4}$
$= 131\frac{1}{4}\,\text{cm}^2$

OR Area $= 7 \times (7 + 11\frac{3}{4}) = 7 \times 18\frac{3}{4}$
$= 126 + 5\frac{1}{4} = 131\frac{1}{4}\,\text{cm}^2$

7 a No, $4\frac{5}{8}$ rounds to 5, so estimate is $15\,\text{m}^2$

b $13\frac{7}{8}\,\text{m}^2$

c Yes, it will cost her $14 \times 15 = \$210$ which is more than \$200.

8 $4\frac{2}{5} \times 21$ is greater because $3\frac{2}{3} \times 25 = 91\frac{2}{3}$ and

$4\frac{2}{5} \times 21 = 92\frac{2}{5}$

9 **B** $12 \times 3\frac{1}{5} = 38\frac{2}{5}$, **C** $5 \times 7\frac{5}{7} = 38\frac{4}{7}$, **A** $8 \times 4\frac{5}{6} = 38\frac{2}{3}$

10 a i $3 \times 12 = 36$ **ii** $31\frac{1}{2}$

    b i $7 \times 10 = 70$ **ii** $67\frac{1}{2}$

    c i $3 \times 15 = 45$ **ii** $46\frac{1}{4}$

11 3 hours 45 minutes

12 a $2 \times (10 - 6) = 8\,m^2$ **b** $6\frac{9}{20}\,m^2$

13 a $9 \times 8\frac{5}{6} = 79\frac{1}{2}, 6 \times 11\frac{3}{4} = 70\frac{1}{2}, 12 \times 7\frac{4}{15} = 87\frac{1}{5}$

    b $79\frac{1}{2}$ **c** $16\frac{7}{10}$

14 a $240\,cm^3$

    b He is correct.

Volume $= 3\frac{3}{4} \times 12 \times 4\frac{2}{3} = 45 \times 4\frac{2}{3} = 210\ cm^3$

OR Volume $= 3\frac{3}{4} \times 12 \times 4\frac{2}{3} = 3\frac{3}{4} \times 56 = 210\ cm^3$

15 a $x = 12$ **b** $y = 13$

Exercise 7.5

1 a i 2   **ii** 4

      iii 6   **iv** 8

    b i 10   **ii** 12

      iii 14   **iv** 16

2 a i 3   **ii** 6

      iii 9   **iv** 12

    b i 15   **ii** 18

      iii 21   **iv** 24

3 a i 4   **ii** 8

    b i 12   **ii** 16

      iii 28   **iv** 40

4 a 5   **b** 18   **c** 16

5 a 22   **b** 100   **c** 108

6 a 3   **b** 5   **c** 4

7 a $13\frac{1}{3}$ **b** $9\frac{1}{3}$ **c** $15\frac{3}{4}$

    d $19\frac{1}{5}$ **e** $7\frac{5}{7}$

8 a 14   **b** $3\frac{1}{3}$

    c $12\frac{1}{2}$ **d** $8\frac{4}{7}$

9 $22\frac{1}{2}\,m$

10 a $s = 7\frac{1}{5}$ **b** $t = 49\frac{1}{2}$

11 **B** gives the greater answer. **A** $25 \div \frac{6}{7} = 29\frac{1}{6}$

B $25 \div \frac{3}{8} = 66\frac{2}{3}$

12 **A** gives the smaller answer.

A $32 \div \frac{13}{15} = 36\frac{12}{13}$, **B** $35 \div \frac{13}{15} = 40\frac{5}{13}$

13 a $5\frac{1}{6}$

    b add $\frac{1}{3}$ (or add $\frac{2}{6}$)

    c $6\frac{1}{6}, 6\frac{1}{2}, 6\frac{5}{6}$

    d $8\frac{1}{6}$

14 a

x	0	2	4	6	8
y	0	$1\frac{1}{2}$	3	$4\frac{1}{2}$	6

    b Plotting the points from the table and drawing the line $y = \frac{3}{4}x$ from $x = 0$ to $x = 8$

15 a $y = 84°$ **b** $y = 48\frac{9}{10}°$ or $48.9°$

    c If $x = 65°$, $x \div \frac{5}{6} = 78°$ and

$x \div \frac{2}{3} + 15 = 112\frac{1}{2}°$. $78 + 112\frac{1}{2} = 190\frac{1}{2}°$

this is > 180° in a triangle, so is not possible.

Exercise 7.6

1 a $\frac{3}{10}$ **b** $\frac{4}{9}$

    c $\frac{7}{8}$ **d** $\frac{1}{3}$

2 **a** $\dfrac{1}{6}$ **b** $\dfrac{1}{8}$

   c $\dfrac{5}{9}$ **d** $\dfrac{1}{2}$

3 **a** $\dfrac{7}{10}$ **b** $\dfrac{10}{21}$

   c $\dfrac{11}{12}$ **d** $\dfrac{23}{30}$

4 **a** $\dfrac{5}{14}$ **b** $\dfrac{7}{15}$

   c $\dfrac{11}{35}$ **d** $\dfrac{7}{30}$

5 $\dfrac{4}{15}$

6 **a** $\dfrac{3}{4}$ **b** $\dfrac{5}{8}$

   c $\dfrac{9}{10}$ **d** $\dfrac{7}{9}$

   e $\dfrac{5}{6}$ **f** $\dfrac{8}{15}$

   g $\dfrac{5}{9}$ **h** $\dfrac{5}{8}$

   i $\dfrac{3}{20}$ **j** $\dfrac{3}{10}$

   k $\dfrac{4}{9}$ **l** $\dfrac{2}{15}$

7 **a** $\dfrac{9}{20}$ **b** $\dfrac{11}{24}$

   c $\dfrac{14}{45}$ **d** $\dfrac{17}{36}$

   e $\dfrac{21}{22}$ **f** $\dfrac{35}{36}$

   g $\dfrac{3}{10}$ **h** $\dfrac{7}{20}$

   i $\dfrac{11}{21}$ **j** $\dfrac{7}{20}$

   k $\dfrac{13}{55}$ **l** $\dfrac{31}{63}$

8 $\dfrac{19}{56}$

9 **a** **i** 8   **ii** 15

   iii 14   **iv** 28

   v 20   **vi** 25

   b learners' checks

   c learners' answers

10 24 m

11 **a** He has calculated $14 \times \dfrac{1}{5}$ not $14 \div \dfrac{1}{5}$

   b 70

12 **a** 6   **b** 9   **c** $\dfrac{2}{3}$ **d** 0

13 **A** $\left(\dfrac{1}{3}+\dfrac{1}{6}\right)\times\left(\dfrac{5}{8}-\dfrac{3}{8}\right)=\dfrac{1}{8}$

   B $\left(3\dfrac{1}{4}+2\dfrac{3}{4}\right)\div\left(\dfrac{2}{7}+\dfrac{3}{14}\right)=12$

   B $\div$ **A** $=12\div\dfrac{1}{8}=96$

Sofia is correct.

14 $d=5\dfrac{1}{4}$ $[a=\dfrac{2}{3}, b=27, c=13\dfrac{1}{2}]$

Exercise 8.1

1 **a** pentagon, 5 sides

   b hexagon, 6 sides

   c octagon, 8 sides

   d decagon, 10 sides

2 **a** All sides are the same length.

   b *AB* is parallel to *DC* and *AD* is parallel to *BC*.

   c All the angles are 90°.

3 **a** Opposite sides are the same length.

   b *EH* is parallel to *FG* and *EF* is parallel to *HG*.

   c All the angles are 90°.

4 **a** *IJ* is the same length as *IL* and *JK* is the same length as *LK*.

   b None of the sides are parallel.

   c Angle *ILK* is equal to angle *IJK* but angle *LIJ* is not equal to angle *LKJ*.

5 **a** Opposite sides are the same length.

   b *NO* is parallel to *MP* and *PO* is parallel to *MN*.

   c Angle *PMN* is equal to angle *NOP* and angle *MNO* is equal to angle *MPO*.

6 **a** All sides are the same length.

   b Opposite sides are parallel.

   c Angle *TQR* is equal to angle *TSR* and angle *QTS* is equal to angle *QRS*.

7 a F   **b** T

   c F   **d** T

   e T   **f** F

8 A and **ii**, B and **iv**, C and **i**, D and **iii**

9 No. At the moment she could be describing a rectangle or a parallelogram. She needs to add the information that all the angles are 90°.

10 A and **iii**, B and **i**, C and **ii**

11 No. At the moment, he could be describing a normal trapezium or an isosceles trapezium. He needs to say that the shape has two pairs of equal angles.

12 a J   **b** N   **c** L

   d I   **e** H   **f** M

   g K

13 a T   **b** T

   c F   **d** F

   e F   **f** T

14 a (2, 2), (6, 2), (6, 8), (2, 8)

   b (2, 2), (0, 4), (2, 6), (4, 4)

   c (2, 2), (6, 2), (8, 4), (4, 4); other combinations are possible

   d (4, 4), (6, 8), (4, 9), (2, 8); other combinations are possible

   e (2, 6), (6, 8), (4, 9), (2, 8); other combinations are possible

Exercise 8.2

1 a $d = 4\,\text{cm}$ **b** $d = 5\,\text{cm}$

     $C = \pi d$ $C = \pi d$

       $= 3.14 \times 4$ $= 3.14 \times 5$

       $= 12.6\,\text{cm}$ $= 15.7\,\text{cm}$

2 a $d = 8\,\text{cm}$ **b** $d = 6\,\text{cm}$

     $C = \pi d$ $C = \pi d$

       $= 3.142 \times 8$ $= 3.142 \times 6$

       $= 25.14\,\text{cm}$ $= 18.85\,\text{cm}$

3 a $d = 9\,\text{cm}$ **b** $d = 12\,\text{cm}$

     $C = \pi d$ $C = \pi d$

       $= \pi \times 9$ $= \pi \times 12$

       $= 28.27\,\text{cm}$ $= 37.70\,\text{cm}$

4 a $C = \pi d$ **b** $C = \pi d$

       $= 3.14 \times 10$ $= 3.14 \times 2$

       $= 31.4\,\text{cm}$ $= 6.28\,\text{cm}$

5 a 50.27 cm   **b** 29.85 m

   c 62.83 cm   **d** 17.59 m

6 a 11.1 cm   **b** 2.8 m

7 a 3.66 cm   **b** 2.83 m

8 18 mm

9 $C = 10.45\,\text{m} = 1045\,\text{cm}$

   $r = \dfrac{C}{2\pi} = \dfrac{1045}{2 \times \pi} = 166.3169\ldots\text{cm}$

     $= 166\,\text{cm}$ (nearest cm)

10 61.70 cm

11 Marcus is incorrect.

   Perimeter of semicircle =

   $\dfrac{\pi d}{2} + d = \dfrac{\pi \times 8}{2} + 8 = 20.57\,\text{cm}$

   Perimeter of three-quarter circle =

   $\dfrac{3\pi d}{4} + r + r = \dfrac{3 \times \pi \times 6}{4} + 3 + 3 = 20.14\,\text{cm}$

   Perimeter of the semicircle is greater than (not less than) the perimieter of the three-quarter circle because 20.57 cm > 20.14 cm.

12 a 49.19 cm   **b** 40.27 cm

13 a 400 m   **b** 46.56 m

   c 461 m

Exercise 8.3

1 a Faces = 7, Vertices = 10, Edges = 15

   b E = 7 + 10 − 2 = 15

   c Edges = 18

   d Vertices = 11

2 a $30 \div 6 = 5\,\text{cm}$ $24 \div 6 = 4\,\text{cm}$ $12 \div 6 = 2\,\text{cm}$

   b A and **ii**, B and **iii**, C and **i**

3 a $8 \div 2 = 4\,\text{cm}$

Learners' scale drawings should be the following squares with dimensions shown.

Top view: Square of side length $4\,\text{cm}$

Front view: Square of side length $4\,\text{cm}$

Side view: Square of side length $4\,\text{cm}$

b $6 \div 2 = 3\,\text{cm}$

$10 \div 2 = 5\,\text{cm}$

$12 \div 2 = 6\,\text{cm}$

Learners' scale drawings should be the following rectangles with dimensions shown.

The dimensions are given as width by height.

Top view: Rectangle $6\,\text{cm}$ by $5\,\text{cm}$

Front view: Rectangle $6\,\text{cm}$ by $3\,\text{cm}$

Side view: Rectangle $5\,\text{cm}$ by $3\,\text{cm}$

c $8 \div 2 = 4\,\text{cm}$

$11 \div 2 = 5.5\,\text{cm}$

$16 \div 2 = 8\,\text{cm}$

Learners' scale drawings should be the following shapes with dimensions shown.

The dimensions for the rectangles are given as width by height.

Top view: Circle radius $4\,\text{cm}$

Front view: Rectangle $8\,\text{cm}$ by $5.5\,\text{cm}$

Side view: Rectangle $8\,\text{cm}$ by $5.5\,\text{cm}$

4 a **B**   **b** **A**

c **B**   **d** **C**

5 Plan view, front and side elevations are shown on centimetre squared paper.
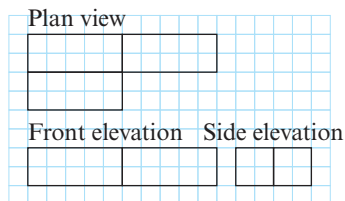
6 Plan view, front and side elevations are shown on centimetre squared paper.
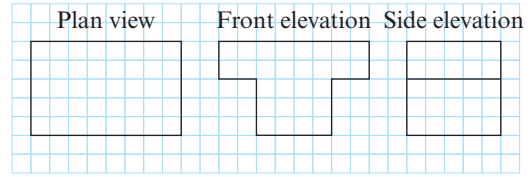
7 Plan view, front and side elevations are shown on centimetre squared paper.
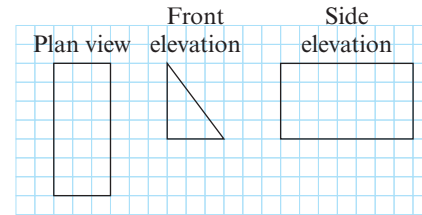
8 Plan view, front and side elevations are shown on centimetre squared paper.
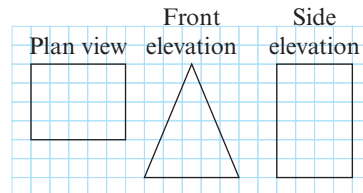
9 Plan view, front and side elevations are shown on centimetre squared paper.
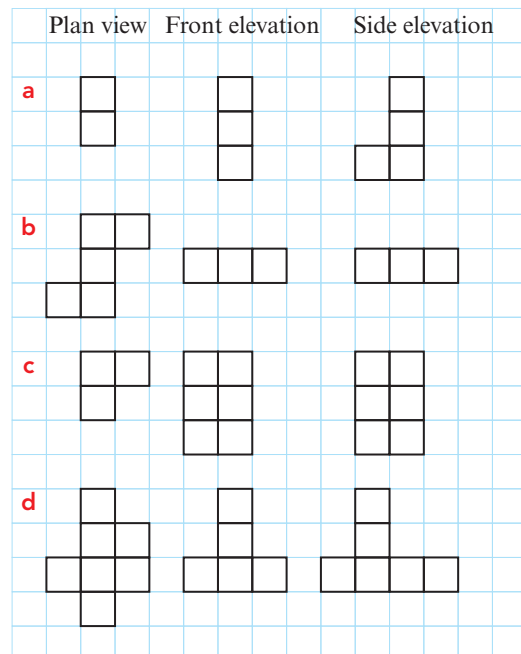
10 Plan view is incorrect. It should be
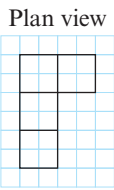
Plan view

She has the front elevation and the side elevation the wrong way round.

11 Plan view, front and side elevations are shown on centimetre squared paper.
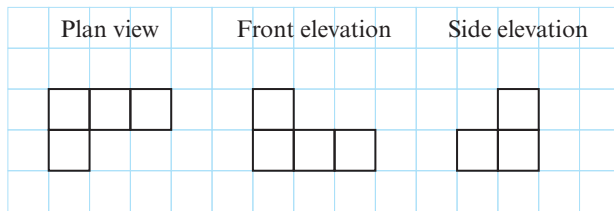
Plan view	Front elevation	Side elevation

12 Plan view, front and side elevations are shown on centimetre squared paper.

Plan view	Front elevation	Side elevation
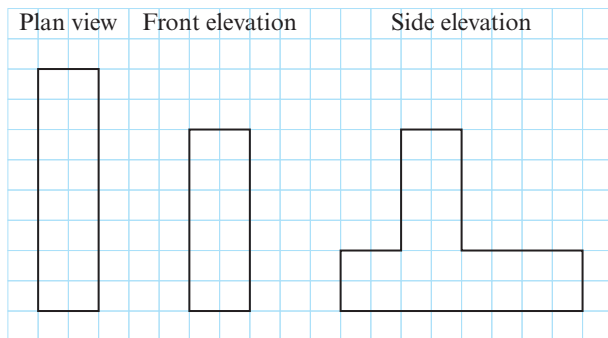
Exercise 9.1

1 a $8+\frac{1}{2}=8\frac{1}{2}, 8\frac{1}{2}+\frac{1}{2}=9, 9+\frac{1}{2}=9\frac{1}{2}$

The term-to-term rule is: add $\frac{1}{2}$

The next two terms are: $9\frac{1}{2}+\frac{1}{2}=10$
$10+\frac{1}{2}=10\frac{1}{2}$

b $8+0.3=8.3, 8.3+0.3=8.6, 8.6+0.3=8.9$

The term-to-term rule is: add 0.3

The next two terms are: $8.9+0.3=9.2$
$9.2+0.3=9.5$

c $5\frac{1}{3}-\frac{1}{3}=5, 5-\frac{1}{3}=4\frac{2}{3}, 4\frac{2}{3}-\frac{1}{3}=4\frac{1}{3}$

The term-to-term rule is: subtract $\frac{1}{3}$

The next two terms are: $4\frac{1}{3}-\frac{1}{3}=4$

$4-\frac{1}{3}=3\frac{2}{3}$

d $9.4-0.4=9, 9-0.4=8.6, 8.6-0.4=8.2$

The term-to-term rule is: subtract 0.4

The next two terms are: $8.2-0.4=7.8$
$7.8-0.4=7.4$

2 **A** and **iii**, **B** and **i**, **C** and **ii**, **D** and **iv**

3 a i add $\frac{1}{4}$ **ii** $2\frac{1}{4}, 2\frac{1}{2}$

b i add $1\frac{1}{2}$ **ii** $15, 16\frac{1}{2}$

c i add 0.2   **ii** 4, 4.2

d i subtract $\frac{1}{2}$ **ii** $8, 7\frac{1}{2}$

e i subtract $\frac{2}{5}$ **ii** $13\frac{2}{5}, 13$

f i subtract 0.25   **ii** 16, 15.75

4 a 2, 2.8, 3.6   **b** $3, 6\frac{1}{2}, 10$

c 10, 8.8, 7.6   **d** $30, 27\frac{4}{5}, 25\frac{3}{5}$

e 0.3, 0.6, 1.2   **f** 18, 9, 4.5

5 a add $1\frac{1}{3}$

b $2, 3\frac{1}{3}, 4\frac{2}{3}, 6, 7\frac{1}{3}, 8\frac{2}{3}, 10, 11\frac{1}{3}$

6 a $6, 7\frac{1}{5}, 8\frac{2}{5}, 9\frac{3}{5}, 10\frac{4}{5}, 12, 13\frac{1}{5}$

b $2, 5\frac{1}{4}, 8\frac{1}{2}, 11\frac{3}{4}, 15, 18\frac{1}{4}, 21\frac{1}{2}$

c $20\frac{3}{4}, 20\frac{1}{2}, 20\frac{1}{4}, 20, 19\frac{3}{4}, 19\frac{1}{2}, 19\frac{1}{4}$

d $40, 39\frac{4}{7}, 39\frac{1}{7}, 38\frac{5}{7}, 38\frac{2}{7}, 37\frac{6}{7}, 37\frac{3}{7}$

e 7, 8.4, 9.8, 11.2, 12.6, 14, 15.4

f 20.4, 19.7, 19, 18.3, 17.6, 16.9

7 a 15.3

b No, after $40\frac{4}{5}$ is 51. Increasing by $10\frac{1}{5}$ each time after 51 will not give 100.

c No, the only fractions in this sequence involve $\frac{1}{3}$ and $\frac{2}{3}$, and not $\frac{1}{2}$

8 a 2, 5, 14   **b** 10, 12, 16

c 6, 10, 12

9 a **A**

b 6th term which is 730

10 a 5, 2, −4   **b** 12, 15, 21

c −8, −2, 1

11 Sequence is: 22, 8, 1, $-2\frac{1}{2}$

12 a 6, 4, 3, 2.5, 2.25, 2.125, 2.0625, 2.03125, 2.015625, 2.0078125

   b They never go to 2 or below.

   c 'The terms in this sequence are all greater than 2'.

   You always divide a number greater than 2 by 2, which gives you a number greater than 1, then you add on 1, which takes you back over 2.

13 a She has divided the 10th term by 2 to get the 5th term. She should have subtracted $2\frac{5}{8}$ five times from the 10th term.

   b $11\frac{1}{4}$

14 57

15 a add 4.2   **b** 46.2   **c** 9.6

Exercise 9.2

1 a 3, 5, 7, . . .   **b** add 2

   c
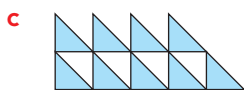
   d Position-to-term rule is: term = 2 × position number + 1

position number	1	2	3	4
term	3	5	7	9
2 × position number	2	4	6	8
2 × position number + 1	3	5	7	9

2 a 5, 8, 11, . . .   **b** add 3

   c
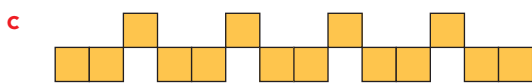
   d Position-to-term rule is: term = 3 × position number + 2

position number	1	2	3	4
term	5	8	11	14
3 × position number	3	6	9	12
3 × position number + 2	5	8	11	14

3 a 8, 12, 16, . . .

   b add 4

   c
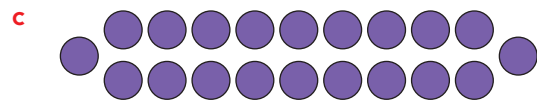
   d Position-to-term rule is: term = 4 × position number + 4

position number	1	2	3	4
term	8	12	16	20
4 × position number	4	8	12	16
4 × position number + 4	8	12	16	20

4 a 7, 10, 13, . . .

   b add 3

   c
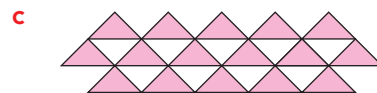
   d Position-to-term rule is: term = 3 × position number + 4

position number	1	2	3	4
term	7	10	13	16
3 × position number	3	6	9	12
3 × position number + 4	7	10	13	16

5 a 2, 6, 10, . . .

   b add 4

   c
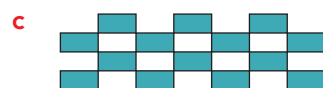
   d Position-to-term rule is: term = 4 × position number − 2

position number	1	2	3	4
term	2	6	10	14
4 × position number	4	8	12	16
4 × position number − 2	2	6	10	14

6 a 2, 5, 8, . . .   **b** add 3

   c
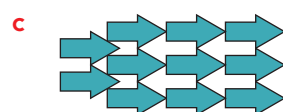
d Position-to-term rule is: term $= 3 \times$ position number $- 1$

position number	1	2	3	4
term	2	5	8	11
$3 \times$ position number	3	6	9	12
$3 \times$ position number $- 1$	2	5	8	11

7 Sami is incorrect.

The sequence of numbers is 5, 7, 9, . . . so the term-to-term rule is 'add 2'.

This means that the position-to-term rule starts with: term $= 2 \times$ position number . . . , so we know Sami's rule is incorrect as it starts with: term $= 5 \times$ position number . . .

The correct position-to-term rule is: term $= 2 \times$ position number $+ 3$

8 **a** term $= 5 \times$ position number $+ 2$

b term $= 15 \times$ position number $- 5$

9 **a** He has the $5 \times$ position number correct, but the $+ 3$ is incorrect as the pattern is taking two hexagons away, not adding an extra three.

b term $= 5 \times$ position number $- 2$

10 73

Sequence is 1, 4, 7, . . . so position-to-term rule is term $= 3 \times$ position number $- 2$

In pattern 25 there are $3 \times 25 - 2 = 73$ rhombuses.

11 **a** 4, 7, 10, . . .   **b** learners' drawings

12 **a** term $= 7 \times$ position number $- 6$

You know the 2nd term is 8, so $? \times 2 - 6 = 8$. Solving this equation gives $? \times 2 = 8 + 6 = 14$, so $? = 14 \div 2 = 7$

b 64

13 28 cm

Sequence of areas is 14, 22, 30, . . .

Term-to-term rule is 'add 8' so position-to-term rule is term $= 8 \times$ position number $+ 6$

Area of 8th rectangle is $8 \times 8 + 6 = 70$

Length of 8th rectangle $= 70 \div 2.5 = 28$ cm

Exercise 9.3

1 **a** 1st term $= 4 \times 1 = 4$ 2nd term $= 4 \times 2 = 8$
3rd term $= 4 \times 3 = 12$ 4th term $= 4 \times 4 = 16$

b 1st term $= 1 + 12 = 13$ 2nd term $= 2 + 12 = 14$
3rd term $= 3 + 12 = 15$ 4th term $= 4 + 12 = 16$

c 1st term $= 2 \times 1 - 1 = 1$
2nd term $= 2 \times 2 - 1 = 3$
3rd term $= 2 \times 3 - 1 = 5$
4th term $= 2 \times 4 - 1 = 7$

d 1st term $= 3 \times 1 + 2 = 5$
2nd term $= 3 \times 2 + 2 = 8$
3rd term $= 3 \times 3 + 2 = 11$
4th term $= 3 \times 4 + 2 = 14$

2 **a** 8, 16, 24, . . . , 80   **b** 2, 7, 12, . . . , 47

c 4, 5, 6, . . . , 13   **d** $-6, -5, -4, . . . , 3$

e 10, 12, 14, . . . , 28 **f** 1, 4, 7, . . . , 28

g 7, 13, 19, . . . , 61   **h** 1, 6, 11, . . . , 46

3 **A** has the smaller value [**A** is $2 \times 8 + 14 = 30$ and **B** is $7 \times 5 - 4 = 31$]

4 $\frac{1}{2} \times 1 + 3 = 3\frac{1}{2}$, $\frac{1}{2} \times 2 + 3 = 4$, $\frac{1}{2} \times 3 + 3 = 4\frac{1}{2}$ and $\frac{1}{2} \times 4 + 3 = 5$

5 **a** 4, $4\frac{1}{2}$, 5, . . . , $7\frac{1}{2}$

b $3\frac{1}{2}$, $7\frac{1}{2}$, $11\frac{1}{2}$, . . . , $31\frac{1}{2}$

c 7.75, 13.75, 19.75, . . . , 49.75

d 2.1, 4.6, 7.1, . . . , 19.6

6 **a** **A** 11, 15, 19, 23   **B** 14, 13, 12, 11

C $10\frac{1}{4}, 10\frac{1}{2}, 10\frac{3}{4}, 11$ **D** $19\frac{2}{3}, 19\frac{1}{3}, 19, 18\frac{2}{3}$

b **A** and **C**

c **B** and **D**

d **B** and **D** are increasing because n is multiplied by a positive number.

A and **C** are decreasing because n is multiplied by a negative number.

7 The sequence 20, 23, 26, 29, 32, . . . is increasing but the nth term expression $23 - 3n$ is for a decreasing sequence because n is multiplied by a negative number.

8 **a** Yes, when $n = 15$, $4 \times 15 + 1 = 61$

b No, when $n = 17$, $3n - 5 = 46$, when $n = 18$, $3n - 5 = 49$, so 48 is not in the sequence.

9 nth term is $3n + 5$

position number (n)	1	2	3	4
term	8	11	14	17
$3 \times n$	3	6	9	12
$3 \times n + 5$	8	11	14	17

10 a $2n + 1$ **b** $3n + 7$

c $4n - 3$ **d** $5n - 1$

11 a 2, 7, 12, 17, . . .   **b** $5n - 3$

c 147

12 a $5n - 20$ **b** $2n - \dfrac{2}{5}$

c $5n - 1.7$

13 She is correct. The nth term expression is $6n - 20$. When $n = 50$, $6 \times 50 - 20 = 300 - 20 = 280$

14 a $8 - n$ **b** $10 - 3n$ **c** $14 - 7n$

15 a -12 **b** -50 **c** -126

16 It's the 15th term when $\mathbf{A} > \mathbf{B}$

The nth term expression for $\mathbf{A}$ is $2\frac{1}{2}n - 15$

14th term $= 20$ 15th term $= 22.5$

The nth term expression for $\mathbf{B}$ is $80 - 4.25n$

14th term $= 20.5$ 15th term $= 16.25$

Exercise 9.4

1 a i

x	0	2	3	$5\frac{1}{2}$
y	4	6	7	$9\frac{1}{2}$

ii
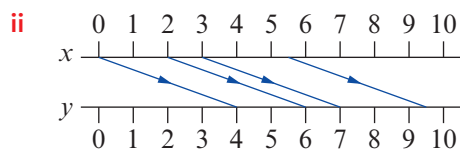
iii $y = x + 4$

b i

x	$1\frac{1}{2}$	3	4	6
y	$4\frac{1}{2}$	9	12	18

ii
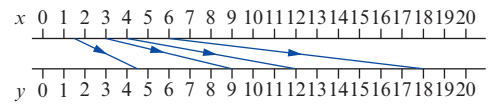
iii $y = 3x$

c i

x	4	6	8	14
y	1	$1\frac{1}{2}$	2	$3\frac{1}{2}$

ii
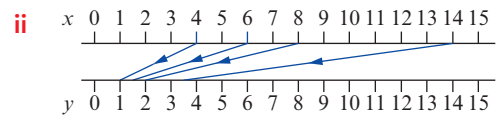
iii $y = \dfrac{x}{4}$

2 a i

x	2	3	4	$4\frac{1}{2}$
y	3	5	7	8

ii $y = 2x - 1$

b i

x	3	6	8	11
y	$2\frac{1}{2}$	4	5	$6\frac{1}{2}$

ii $y = \dfrac{x}{2} + 1$

c i

x	7	9	13	23
y	$2\frac{1}{2}$	3	4	$6\frac{1}{2}$

ii $y = \dfrac{x + 3}{4}$

d i

x	$6\frac{1}{2}$	8	11	$13\frac{1}{2}$
y	6	12	24	34

ii $y = 4(x - 5)$

3 a

x	2	$5\frac{1}{2}$	8	16
y	14	$17\frac{1}{2}$	20	28

b

x	1	3	4	7
y	5	15	20	35

c

x	5	12	17	30
y	3	$6\frac{1}{2}$	9	$15\frac{1}{2}$

4 a i

x	10	$15\frac{1}{4}$	18	$20\frac{1}{3}$
y	1	$6\frac{1}{4}$	9	$11\frac{1}{3}$

ii

x	12	30	39	72
y	2	5	$6\frac{1}{2}$	12

iii

x	20	30	40	100
y	0	2	4	16

iv

x	2	$2\frac{1}{2}$	4	$7\frac{1}{2}$
y	20	21	24	31

b i $y = x - 9$

ii $y = \frac{x}{6}$

iii $y = \frac{x}{5} - 4$

iv $y = 2(x + 8)$

5 a
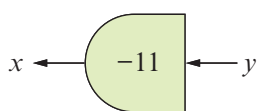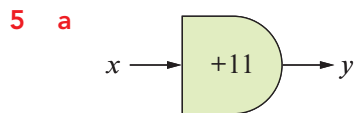
equation: $y = x + 11$

inverse function equation: $x = y - 11$

b
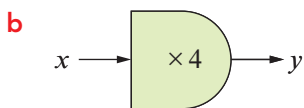
equation: $y = 4x$

inverse function equation: $x = \frac{y}{4}$

c
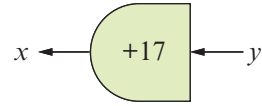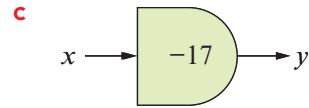
equation: $y = x - 17$

inverse function equation: $x = y + 17$

d
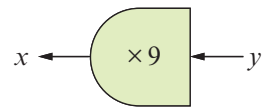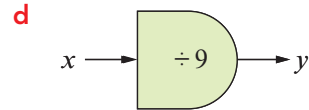
equation: $y = \frac{x}{9}$

inverse function equation: $x = 9y$

e
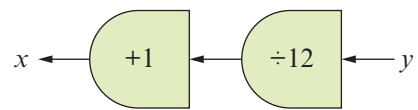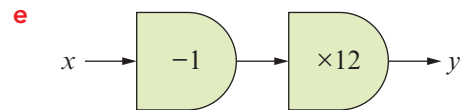
equation: $y = 12(x - 1)$

inverse function equation: $x = \frac{y}{12} + 1$

f
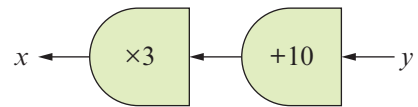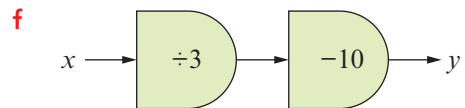
equation: $y = \frac{x}{3} - 10$

inverse function equation: $x = 3(y + 10)$

6 **A** and **ii**, **B** and **v**, **C** and **i**, **D** and **vi**, **E** and **iii**, **F** and **iv**

7 a i $y = \frac{x - 4}{2}$

ii $x = 2y + 4$

b

x	−6	−3	20	31
y	−5	$-3\frac{1}{2}$	8	$13\frac{1}{2}$

8 a i $y = 6x + 8$

ii $x = \dfrac{y-8}{6}$

b

x	−5	$-2\frac{1}{2}$	$2\frac{1}{2}$	$6\frac{1}{2}$
y	−22	−7	23	47

9 a i

$x \longrightarrow \boxed{-4} \longrightarrow y$

ii

$x \longrightarrow \boxed{\div 4} \longrightarrow y$
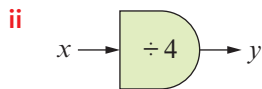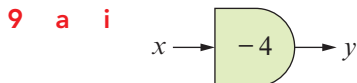
b i $y = x - 4$

ii $y = \dfrac{x}{4}$

10 Using Arun's formula: when $x = -2$,
$y = 3 \times -2 + 2\frac{1}{2} = -3\frac{1}{2}$. This is correct in the table.

When $x = 4$, $y = 3 \times 4 + 2\frac{1}{2} = 14\frac{1}{2}$. This is incorrect in the table, so Arun is wrong.

Using Sofia's formula: when $x = -2$,
$y = 2\frac{1}{2} \times -2 + 1\frac{1}{2} = -3\frac{1}{2}$. This is correct in the table. When $x = 4$, $y = 2\frac{1}{2} \times 4 + 1\frac{1}{2} = 11\frac{1}{2}$.
This is correct in the table. When $x = 7$,
$y = 2\frac{1}{2} \times 7 + 1\frac{1}{2} = 19$. This is correct in the table, so Sofia is correct.

11 $y = 5x - 4$ and learners' explanations

12 $y = 8x - 20$ and learners' methods

Exercise 10.1

1 a 36 kg   **b** 156 kg   **c** 192 kg

2 a $28   **b** $12   **c** $68

3 a 220 km   **b** 250 km   **c** 350 km

4 a 7.5 hours   **b** 12 hours   **c** 3 hours

5 a 322   **b** 518   **c** 658

6 a 5632   **b** 2432   **c** 512

7 a 62.5%   **b** 160%

8 a 38.5%   **b** 260%

9 $92

10

Original value	New value	Absolute change	Percentage change
600	700	100	16.7% increase
75	40	35	46.7% decrease
36	100	64	178% increase
43	12	31	72.1% decrease
250	175	75	30% decrease
90	234	144	160% increase

11 a The missing numbers are 840, 756, 378

b 37% decrease

12 a 10.4% and 11.6%

b $100

c 20.8%

13 a i $30   **ii** 12.2%

b i $30   **ii** 71.4%

c $60

14 a 2694

b 116% or 115.9% to 1 d.p.

c approximately 10 838

15 No: 1970 to 1990 is 44.0% increase; 1990 to 2010 is 30.6% increase

16 a $80 + $80 × 50% = $80 + $40 = $120;
$80 + $80 × 25% = $100 and then
$100 + $100 × 20% = $120

b $80 − $80 × 50% = $40; $80 − $80 × 25% = $60 and $60 − $60 × 20% = $48; they are not the same

Exercise 10.2

1 a 1.05   **b** 1.95

c 2.32   **d** 3

2 a 0.86   **b** 0.33   **c** 0.03

3 **a** $96   **b** 152 kg

   c 448 cm   **d** 1200 mg

4 **a** 350   **b** 325   **c** 175

5 **a** 60% increase   **b** 8% increase

   c 300% increase   **d** 40% decrease

   e 1% decrease   **f** 93% decrease

6 **a** 384   **b** 240

   c A decrease of 37.5% reverses the effect of a 60% increase

7 **a** 162.5%   **b** 62.5%

8 **a** 72(.1)%   **b** 28% or 27.9%

9 **a** 250%   **b** 150%

10 $320 \times 1.1 = 352$ and then $352 \times 1.5 = 528$; $320 \times 1.65 = 528$ and this is the same answer.

11 The numbers clockwise from 120 are 300, 240, 96, 25%

12 **a** 175% increase

   b the largest is 1960 to $1970 = 30.7\%$

   c the smallest is 2000 to $2010 = 14.2\%$

   d learners' own choices of multiplier, with justification

13 **a** 25%, 20%, 16.7%   **b** 8.3%, 9.1%, 10%

   c 972 and 1749.6

14 first year $\$40\,000 \times 0.8 = \$32\,000$; second year $\$32\,000 \times 0.8 = \$25\,600$; third year $\$25\,600 \times 0.8 = \$20\,480$; fourth year $\$20\,480 \times 0.8 = \$16\,384$. This is a drop in value of $\$23\,616$; $\frac{23\,616}{40\,000} \times 100 = 59.04\%$ which is approximately 60%

Exercise 11.1

1 **a** **i** $25   **ii** $40

   b $h = 5n + 10$

2 **a** $44   **b** $y = 8t + 4$

3 **a** **i** $2315   **ii** $4565

   b $r = 750m + 65$

4 **a** **i** 25   **ii** 19

   b $L = 37 - 6t$

5 **a** 15 m   **b** 2 m   **c** 3 m

6 **a** **i** 24   **ii** 18   **iii** 14

   b 12

7 **a** 45   **b** 69   **c** $m = 2w + 5$

8 **a** 9

   b The girl is 6 years less than 3 times the boy's age (or an equivalent statement).

9 **a** 30 cm   **b** $p = 2x + 18$ **c** $a = 2x + 20$

10 all of them

11 learners' own answers

12 **a** 7 days is $45 and 14 days is $80 and $2 \times \$45 = \$90$

   b $5

Exercise 11.2

1 **a**

x	−2	−1	0	1	2	3	4	5
y	−6	−4	−2	0	2	4	6	8

   b graph plotted from table

2 **a**

x	−3	−2	−1	0	1	2	3
y	−5	−2	1	4	7	10	13

   b graph plotted from table

3 **a**

x	−2	−1	0	1	2	3	4	5	6
y	7	6	5	4	3	2	1	0	−1

   b graph plotted from table

4 **a** 11

   b

x	−3	−2	−1	0	1	2	3
y	−13	−9	−5	−1	3	7	11

   c graph plotted from table

5 **a** $6 - 2 \times -2 = 6 - -4 = 10$

   b

x	−2	−1	0	1	2	3	4
y	10	8	6	4	2	0	−2

   c graph plotted from table

   d **A** (−4, 14), **D** (10, −14) and **E** (−10, 26)

6 a

x	−2	−1	0	1	2	3	4
y	−20	−5	10	25	40	55	70

   b graph plotted from table

   c at (0, 10)

7 a

x	−20	−10	0	10	20	30	40
$60-x$	80	70	60	50	40	30	20

   b graph plotted from table

   c at (0, 60)

   d at (60, 0)

8 a

x	−2	−1	0	1	2	3	4
$5x+20$	10	15	20	25	30	35	40

   b graph plotted from table

   c

x	−2	−1	0	1	2	3	4
$30-5x$	40	35	30	25	20	15	10

   d graph plotted from table on same axes as graph for **d**, intersection at (1, 25)

   e at (1, 25)

9 a (5, 115)

   b (8, 190)

   c (−2, −60)

   d (1, 15)

   e (4, 90)

10 a $p = 4x+6$

   b

x	1	2	3	4	5	6	7
p	10	14	18	22	26	30	34

   c graph plotted from table

   d 3.5 cm

11 a $y = 30t+50$

   b

t	1	2	3	4	5
y	80	110	140	170	200

   c graph plotted from table

   d 4.5 hours

12 a

m	0	1	2	3	4	5	6
T	70	64	58	52	46	40	34

   b graph plotted from table

   c It is a straight line sloping downwards

   d After 2.5 minutes

13 a graph plotted through points (1, 14), (2, 18), (3, 22), (4, 26), (5, 30), (6, 34), (7, 38)

   b There is a fixed charge of $10.

   c There is a charge per day of $4.

Exercise 11.3

1 a

x	−4	−3	−2	−1	0	1	2	3	4
$x+2$	−2	−1	0	1	2	3	4	5	6

   b and **c** graph plotted from table, with addition of $y = x$ line, on same axes

   d the gradient of each line is 1

2 a

x	−4	−3	−2	−1	0	1	2	3	4
$2x-1$	−9	−7	−5	−3	−1	1	3	5	7

   b and **c** graph plotted from table, with addition of $y = 2x$ line, on same axes

   d the gradient of each line is 2

   e −1 and 0

3 a

x	−4	−3	−2	−1	0	1	2	3	4
$5x$	−20	−15	−10	−5	0	5	10	15	20
$5x+10$	−10	−5	0	5	10	15	20	25	30

   b graphs plotted from table, on same axes

   c the gradient of each line is 5

   d 0 and 10

4 a

x	−3	−2	−1	0	1	2	3
$x+3$	0	1	2	3	4	5	6

   b

x	−3	−2	−1	0	1	2	3
$2x+3$	−3	−1	1	3	5	7	9

c graphs plotted from tables in **a** and **b**, on same axes

d 1 and 2

e the y-intercept of each line is 3

5 a
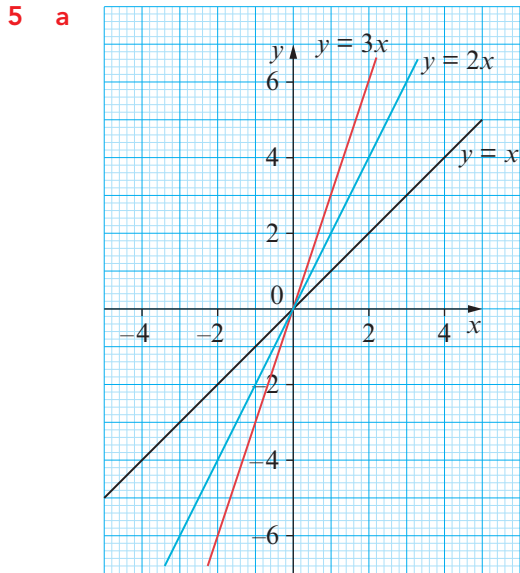
b gradient of $y = x$ is 1; gradient of $y = 2x$ is 2; gradient of $y = 3x$ is 3

6 a

x	−2	−1	0	1	2	3	4
$-x+7$	9	8	7	6	5	4	3

b

x	−2	−1	0	1	2	3	4
$-x+2$	4	3	2	1	0	−1	−2

c graphs plotted from tables in **a** and **b**, on same axes

d both are −1   **e** 7 and 2

f at (7, 0) and (2, 0)

7 a −5   **b** (0, 10)

c (2, 0)   **d** $y = -5x$

8 a

x	−2	−1	0	1	2	3
$10x+20$	0	10	20	30	40	50

b and **c**
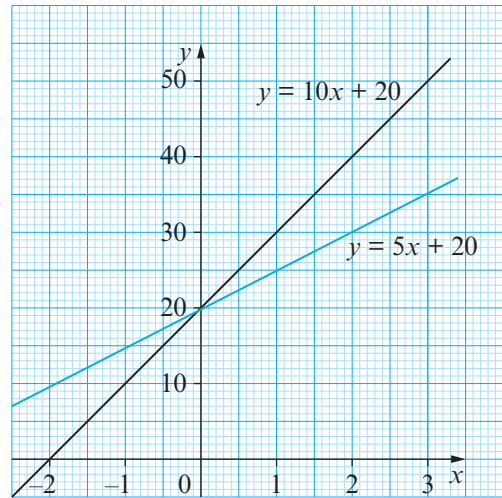
d gradients are 10 and 5; intercepts are both 20

9 a A is $y = x+6$, B is $y = 2x+6$, C is $y = -x+6$, D is $y = -2x+6$

b A at (−6, 0), B at (−3, 0), C at (6, 0) and D at (3, 0)

10 a

x	0	1	2	3	4	5	6
$18-3x$	18	15	12	9	6	3	0

b graph plotted from table

c the tank is empty

d there was initially 18 litres in the tank

e the gradient is −3 which shows that 3 litres flows out every hour

11 a line B

b A is $y = 10x+30$ and C is $y = 10x+10$

c $y = 10x-10$

12 a a possible table:

n	1	2	3	4	5	6	7	8
c	150	250	350	450	550	650	750	850

b graph plotted from learners' own tables

c the gradient is 100 because the cost of each night is $100

Exercise 11.4

1 a 0.5 km   **b** 20 minutes

c 1 km

d They meet when they have walked 2 km, 40 minutes after Zara started.

2 a bucket 1 has 5 litres and bucket 2 has 1 litre

b bucket 1 has 6 litres and bucket 2 has 4 litres

c 1 **d** 3

e after 2 minutes

3 a 30 minutes **b** 5 km

c 09:20

d Sofia. Valid reasons are Sofia's graph is steeper, or Sofia only took 20 minutes but Zara took 30 minutes.

4 a 40 litres **b** 8 litres

c 25 litres

d

Hours	0	1	2	3	4
Litres	25	23	21	19	17

e 2 litres

5 a plumber B

b A $20 and B $10

c

Hours	0	1	2	3	4	5
Cost	20	25	30	35	40	45

d $5 **e** $8

6 a 40 seconds **b** 50 m

c 30 m

d Arun and Marcus meet after 50 seconds when they are 250 m from Arun's starting point.

7 a $390

b when there are more than 40 people

c

People	0	10	20	30	40	50
Cost ($)	300	330	360	390	420	450

d i $300 **ii** $3

e i $100 **ii** $8

8 a X 10 cm and Y 26 cm

b both lines are straight

c X 5 cm and Y 3 cm

d $y = 3x + 26$

e after 8 weeks

9 a 13 °C

b 10 °C, 15 °C, 4 °C

10 a A is $260 and B is $220

b the line meets the y-axis at 180; the charge increases by $20 per day so the gradient is 20

c $y = 30x + 100$

Exercise 12.1

1 a 1 : 3 **b** 1 : 9

c 1 : 8 **d** 1 : 4

e 1 : 3 **f** 5 : 1

g 10 : 1 **h** 9 : 1

i 70 : 1 **j** 4 : 1

2 a 3 : 4 **b** 2 : 3

c 4 : 5 **d** 3 : 5

e 5 : 7 **f** 4 : 3

g 3 : 2 **h** 9 : 8

i 4 : 3 **j** 5 : 3

3 a 1 : 5 : 6 **b** 2 : 3 : 4

c 4 : 3 : 5

4 a 5 : 6 : 2 **b** 4 : 1 : 8

c 12 : 3 : 5

5 a She has split the ratio (of 3 numbers) into two ratios (of two numbers) and simplified them separately instead of simplifying the one ratio (of 3 numbers).

b 3 : 10 : 5

6 a 1 : 4 **b** 40 : 3

c 21 : 10 **d** 4 : 1

e 10 : 7 **f** 7 : 2

7 Arun is wrong. The ratio of walnuts to dates is 600 : 1000 = 3 : 5

8 a 20 : 40 : 3 **b** 20 : 11 : 4

c 90 : 3 : 1 **d** 9 : 3 : 40

e 2 : 30 : 1 **f** 60 : 11 : 50

9 They are both wrong. The ratio of butter to sugar to flour is 9 : 11 : 22

10 a one mug [one cup holds 225 ml, one mug holds 250 ml]

b one bag of red rice [bag red rice 320 g, bag brown rice 305 g]

c Jules can text more quickly [Jules 1 word in 3 seconds, Sion 1 word in 3.5 seconds]

11 a 1 : 5   **b** 1 : 2

c 1 : 4   **d** 1 : 5

e 2 : 1   **f** 7 : 5

g 5 : 12   **h** 3 : 2

i 1 : 6 : 20

12 a learners' answers

Example: Route 3 takes less time than route 2, but the number in the ratio is higher.

b 55 mins is not 0.55 hours and 1 hour 10 mins is not 1.1 hours.

c 11 : 18 : 14

13 a i 1 : 2   **ii** 0.5 : 1

b i 1 : 4   **ii** 0.25 : 1

c i 1 : 5   **ii** 0.2 : 1

d i 1 : 8   **ii** 0.125 : 1

14 learners' answers.

Examples:

a

Length of side	Length of diagonal	Ratio of lengths side : diagonal	Ratio of lengths side : diagonal in the form 1 : n
3 cm	4.2	3 : 4.2	1 : 1.4
4 cm	5.7	4 : 5.7	1 : 1.4
5 cm	7.1	5 : 7.1	1 : 1.4

b They are the same. 1 : 1.4

c $8 \times 1.4 = 11.2$ cm

d $14 \div 1.4 = 10$ cm

Exercise 12.2

1 Total number of parts: $4 + 1 + 3 = 8$

Value of one part: $\$72 \div 8 = 9$

Ali gets: $4 \times 9 = \$36$

Bob gets: $1 \times 9 = \$9$

Carl gets: $3 \times 9 = \$27$

2 a $10 : $30 : $50

b $60 : $80 : $100

c $300 : $500 : $200

d $125 : $50 : $175

3 a, b i G : H : I = $20 : $40 : $50
Check: $20 + 40 + 50 = \$110$

ii G : H : I = $30 : $60 : $75
Check: $30 + 60 + 75 = \$165$

iii G : H : I = $64 : $128 : $160
Check: $64 + 128 + 160 = \$352$

4 a i 60   **ii** 48

iii 132

b $60 + 48 + 132 = 240$

5 40º : 60º : 80º

6 a i 35   **ii** 7

iii 14

b i 24   **ii** 6

iii 12

7 Zara $125, Sofia $300, Arun $100 and Marcus $225

8 $6 + 8 + 11 + 13 = 38$, $4750 \div 38 = 125$

$13 \times 125 = \$1625$

9 22 oranges

10 Tatiana pays $10 000, Lucia pays $70 000 and Gianna pays $40 000.

Ratio T : L : G = 1 : 7 : 4, $1 + 7 + 4 = 12$, $\$210\,000 \div 12 = \$17\,500$ per part

Gianna receives $4 \times \$17\,500 = \$70\,000$

Profit for Gianna $= \$70\,000 - \$40\,000 = \$30\,000$

Gianna is correct.

11 a X : Z : M = $1100 : $825 : $1375

b $375

c Zane, he lost $225 [Xiu lost $300 and Mike lost $375]

12 $2 + 3 + 7 = 12$, $24 \div 12 = 2$

number of 50 cent coins $2 \times 2 = 4$
value $= 4 \times 50$ cent $= $2

number of 25 cent coins $3 \times 2 = 6$
value $= 6 \times 25$ cent $= $1.50

number of 10 cent coins $7 \times 2 = 14$
value $= 14 \times 10$ cent $= $1.40

Total value $= 2 + 1.5 + 1.40 = $4.90

13 accept equivalent values

Size of tin	Blue	Yellow	Green
1 litre	400 ml	550 ml	50 ml
1.5 litres	600 ml	825 ml	75 ml
2.5 litres	1 l	1.375 l	125 ml

14 Salim is wrong, the largest angle is 120° more than the smallest angle, not 110°.

Angles are 30°, 120°, 150°, 60°
Largest − smallest $= 150° − 30° = 120°$

Exercise 12.3

1 Arshan 1 : 2 and Oditi 1 : 3   Oditi has the darker paint.

2 Jake 1 : 5 and Razi 1 : 3   Jake has the darker paint.

3 a total number of parts $2 + 3 = 5$

  i fraction that are brazil nuts $= \dfrac{2}{5}$

  ii fraction that are almonds $= \dfrac{3}{5}$

b total number of parts $5 + 4 = 9$

  i fraction that are tennis balls $= \dfrac{5}{9}$

  ii fraction that are footballs $= \dfrac{4}{9}$

c total number of parts $1 + 9 = 10$

  i fraction that are onions $= \dfrac{1}{10}$

  ii fraction that are potatoes $= \dfrac{9}{10}$

4 a Orange sunset: total number of parts $=$
$3 + 2 = 5$ fraction orange $= \dfrac{3}{5}$

Orange flame: total number of parts $=$
$5 + 3 = 8$ fraction orange $= \dfrac{5}{8}$

b Orange sunset: fraction orange $=$
$\dfrac{3}{5} = \dfrac{3 \times 8}{5 \times 8} = \dfrac{24}{40}$

Orange flame: fraction orange $=$
$\dfrac{5}{8} = \dfrac{5 \times 5}{8 \times 5} = \dfrac{25}{40}$

c Orange flame

d Orange flame as it has a greater fraction that is orange and a smaller fraction that is white compared to orange sunset.

5 a total number of parts $= 5 + 4 = 9$
fraction boys $= \dfrac{5}{9}$

b boys in the club $=$ fraction boys $\times 45 =$
$\dfrac{5}{9} \times 45 = 25$

6 a fruit drink **A**: total number of parts $=$
$2 + 7 = 9$ fraction pear juice $= \dfrac{7}{9}$
fruit drink **B**: total number of parts $=$
$5 + 13 = 18$ fraction pear juice $= \dfrac{13}{18}$

b fruit drink **A**: fraction pear juice $=$
$\dfrac{7}{9} = \dfrac{7 \times 2}{9 \times 2} = \dfrac{14}{18}$
fruit drink **B**: fraction pear juice $= \dfrac{13}{18}$

c **A**

d **A** as it has the greater fraction that is pear juice.

7 a $\dfrac{3}{10}$ **b** 12

8 **D** 72, as it's the only number divisible by 8.

9 39

10 a 3 : 1   **b** 4 : 5

c 7 : 3

11 a Banana yellow $\frac{3}{8}$ white, Mellow yellow $\frac{5}{12}$ white

b Banana yellow $\frac{3}{8} = \frac{9}{24}$ white, Mellow yellow $\frac{5}{12} = \frac{10}{24}$ white

Mellow yellow is lighter as it has a greater fraction of white paint.

12 a Gavin, $\frac{7}{9}$ is pineapple juice. Matt, $\frac{10}{13}$ is pineapple juice.

b Gavin, $\frac{7}{9} = \frac{91}{117}$ is pineapple juice. Matt, $\frac{10}{13} = \frac{90}{117}$ is pineapple juice.

Gavin's has the higher proportion of pineapple juice as the fraction of pineapple juice is greater.

13 Li. Example solution:

Li has 32 magazines and 12 puzzle books which is equivalent to 48 magazines and 18 puzzle books.

Su has 45 magazines and 18 puzzle books.

Li has the greater proportion of magazines because 48 > 45 magazines for the same number of puzzle books.

14 'Clothes 2 Keep'. Example solution:

'Clothes 4 U' has 24 coats and 60 jumpers for sale, which is equivalent to 40 coats and 100 jumpers.

'Clothes 2 Keep' has 40 coats and 95 jumpers for sale.

'Clothes 2 Keep' has the greater proportion of coats because 95 < 100 jumpers for the same number of coats.

15 a $\frac{2}{7}$ **b** 600 kg

16 8 women

Exercise 13.1

1 a 0.85   **b** 0.6

2 a 95%   **b** 85%

c 20%

3 a $\frac{3}{4}$ **b** $\frac{7}{8}$

4 a $\frac{1}{2}$ **b** $\frac{1}{4}$

5 a

Second spin			
G	RG	YG	GG
Y	RY	YY	GY
R	RR	YR	GR
	R	Y	G

First spin

b $\frac{1}{9}$ **c** $\frac{2}{9}$

d $\frac{4}{9}$ **e** $\frac{5}{9}$

6 a

	1	2	3	4	5	6	7
H	1H	2H	3H	4H	5H	6H	7H
T	1T	2T	3T	4T	5T	6T	7T

b $\frac{1}{14}$ **c** $\frac{3}{14}$ **d** $\frac{6}{14}$ or $\frac{3}{7}$

7 a 567, 576, 657, 675, 756, 765

b i $\frac{4}{6} = \frac{2}{3}$ **ii** $\frac{2}{6} = \frac{1}{3}$

iii $\frac{4}{6} = \frac{2}{3}$

8 a

3	4	5	6	7
2	3	4	5	6
1	2	3	4	5
	1	2	3	4

b i $\frac{2}{12} = \frac{1}{6}$ **ii** $\frac{3}{12} = \frac{1}{4}$

iii $\frac{6}{12} = \frac{1}{2}$ **iv** $\frac{6}{12} = \frac{1}{2}$

9 a

6	6	12	18	24	30	36
5	5	10	15	20	25	30
4	4	8	12	16	20	24
3	3	6	9	12	15	18
2	2	4	6	8	10	12
1	1	2	3	4	5	6
	1	2	3	4	5	6

b 6 or 12; both have probability of $\frac{4}{36} = \frac{1}{9}$

c $\frac{11}{36}$ **d** $\frac{9}{36} = \frac{1}{4}$

10 Here is a table of outcomes.

6	5	4	3	2	1	0
5	4	3	2	1	0	1
4	3	2	1	0	1	2
3	2	1	0	1	2	3
2	1	0	1	2	3	4
1	0	1	2	3	4	5
	1	2	3	4	5	6

Here are the probabilities.

Difference	0	1	2
Probability	$\frac{6}{36} = \frac{1}{6}$	$\frac{10}{36} = \frac{5}{18}$	$\frac{8}{36} = \frac{2}{9}$

Difference	3	4	5
Probability	$\frac{6}{36} = \frac{1}{6}$	$\frac{4}{36} = \frac{1}{9}$	$\frac{2}{36} = \frac{1}{18}$

11 a 34, 35, 36; 43, 45, 46; 53, 54, 56; 63, 64, 65

   b $\frac{6}{12} = \frac{1}{2}$

   c 345, 346, 354, 356, 364, 365; 435, 436, 453, 456, 463, 465; 534, 536, 543, 546, 563, 564; 634, 635, 643, 645, 653, 654

   d $1 - \frac{6}{24} = \frac{18}{24} = \frac{3}{4}$

   e There are 24. They are all the 3-digit numbers with the fourth digit added as a final digit.

   f $\frac{23}{24}$

12 a

First spin	Second spin	Outcome
	R	RR
R	G	RG
	B	RB
	R	GR
G	G	GG
	B	GB
	R	BR
B	G	BG
	B	BB
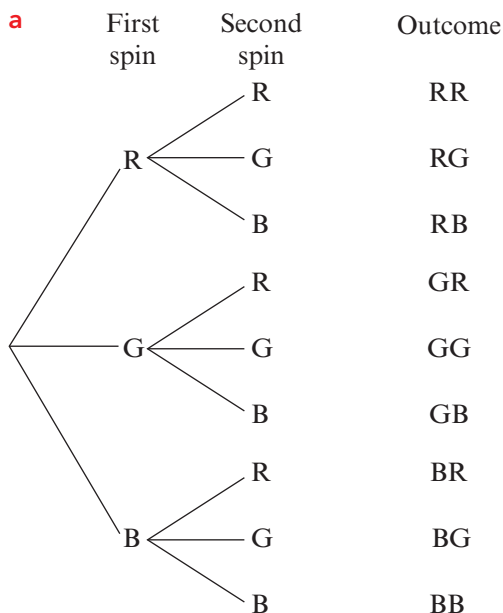
b   **i** $\frac{1}{9}$ **ii** $\frac{1}{9}$

   iii $\frac{8}{9}$ **iv** $\frac{5}{9}$

13 There are $6 \times 8 = 48$ outcomes, as shown on this diagram.
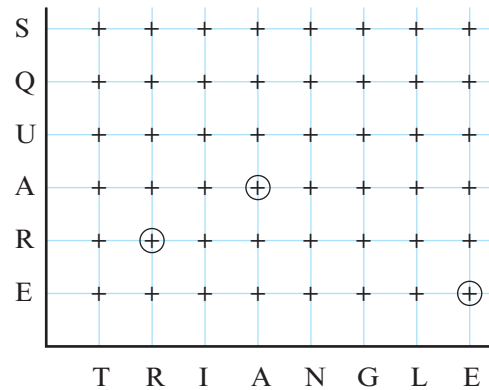
Two letters the same can happen in 3 ways, AA, RR or EE so the probability is $\frac{3}{48} = \frac{1}{16}$

Exercise 13.2

1 a orange 0.425, pink 0.35, white 0.225

   b orange 0.5, pink 0.333, white 0.167

2 a 0.15   **b** 0.125   **c** 0.1875

   d The theoretical probability is 0.167 and the last experimental probability is closest to this.

   e 20 throws = 0.35; 40 throws = 0.5; 80 throws = 0.475; the experimental probability for 40 throws is closest to 0.5

3 a yellow = 0.2; blue = 0.233; green 0.367; red = 0.2

   b The theoretical probability for each colour is 0.25; the value for green seems large but this is because the sample size is quite small. The values for yellow, blue and red are all quite similar.

4 a   **i** 0.2075   **ii** 0.6275

   iii 0.3525

   b   **i** 0.2   **ii** 0.6

   iii 0.4

   c The experimental probabilities are quite close to the theoretical probabilities so we can assume that the spinner is fair.

5 **a** and **b** probabilities in a table

Heads	0	1	2	3
Experimental	0.118	0.360	0.384	0.138
Theoretical	0.125	0.375	0.375	0.125

    c The probabilities are very similar.

    d

Heads	0	1	2	3
Experimental	0.128	0.376	0.364	0.132
Theoretical	0.125	0.375	0.375	0.125

    e yes

6 **a** green 0.3025; gold 0.5775; black 0.12

    b a sensible conjecture would be 3 green faces, 6 gold faces and 1 black face

    c based on 1000 throws, green 0.293; gold 0.603; black 0.104

    d no; the theoretical probabilities for 1000 throws are even closer to 0.3, 0.6 and 0.1, which suggests 3 green faces, 6 gold faces and 1 black face

7 Learners' own work. The more throws are made, the more evidence there is to support a conclusion. In part **a** theoretical probabilities are 0.5; in part **b** probabilities for 0, 1 or 2 heads can be compared.

Exercise 14.1

1 **A** and **iii**, **B** and **iv**, **C** and **i**, **D** and **v**, **E** and **ii**

2 **a** 115°   **b** 240°   **c** 060°

    d 325°   **e** 132°

3 **a** 025°   **b** 155°   **c** 200°

4 Learners' accurate diagrams to show these bearings of Y from X.

    a 065°   **b** 105°

    c 230°   **d** 350°

5 Ivan is incorrect; learners' explanations

    Example: The bearing of B from A is 208°. He has given the angle from North measured anticlockwise not clockwise.

6 **a** 057°   **b** 237°

7 **a** 110°   **b** 045°   **c** 155°

    d 275°   **e** 330°

8 **a** **Ai** 036°   **Aii** 216°

        Bi 124°   **Bii** 304°

        Ci 073°   **Cii** 253°

    b Answer to **ii** = answer to **i** + 180°

    c **Di** 083°   **Dii** 263°

        Ei 137°   **Eii** 317°

        Fi 022°   **Fii** 202°

9 **a** **Ai** 238°   **Aii** 058°

        Bi 288°   **Bii** 108°

        Ci 261°   **Cii** 081°

    b Answer to **ii** = answer to **i** − 180°

    c **Di** 232°   **Dii** 052°

        Ei 336°   **Eii** 156°

        Fi 198°   **Fii** 018°

10 **a** 050°   **b** 230°

    c Explanations showing the calculation of angles as shown.

$$180 - 70 = 110°$$
70° (Alternate angles)
$$360 - 70 = 290°$$
120 − 50 = 70°
50° (Alternate angles)
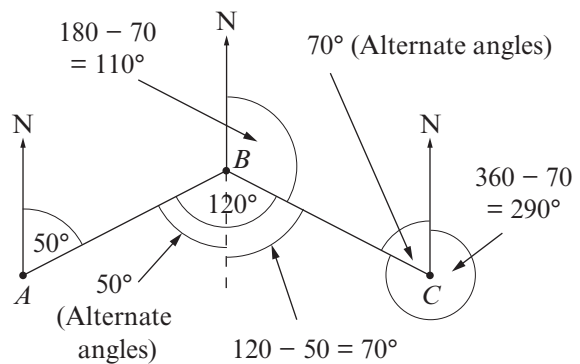
    d Marcus is incorrect. The bearing of B from C is 110 + 180 = 290°.

    e No, it is not possible to work out the bearing of C from A.

    Learners' explanations. Example: You don't know the distances from A to B and B to C.

11 **a** 090°

    b **i** 150°   **ii** 210°   **iii** 270°

        iv 330°   **v** 030°

c Learners' working. Example: *ABCD* is an isosceles trapezium with $\angle ABC = \angle BCD = 120°$

$360 - 2 \times 120 = 120$ and $120 \div 2 = 60°$, so $\angle CDA = \angle BAD = 60°$

So bearing to get from *A* to *D* is $90 + 60 = 150°$.

d Zara is correct.

Learners' working. Example: *ABE* is a right angled triangle. $\angle BAE = 90°$

AFE is an isosceles triangle.
$\angle AEF = \frac{180 - 120}{2} = 30°$

Also $\angle FEB = 60°$ (isosceles trapezium as in part c). $\angle AEB = 60 - 30 = 30°$, so bearing is 030°

Exercise 14.2

1 **a** *A* is $(-2, 3)$ *B* is $(4, 3)$ The midpoint of *AB* is $(1, 3)$

b *C* is $(4, -2)$ *D* is $(4, -6)$ The midpoint of *CD* is $(4, -4)$

c *E* is $(0, -5)$ *F* is $(-6, -5)$ The midpoint of *EF* is $(-3, -5)$

2 **a** **B** $(2, 5)$ **b** **C** $(8, 16)$

c **A** $(5, 1)$ **d** **C** $(6, 15)$

3 Sofia is incorrect, the midpoint is at $(3.5, 3)$ not $(3, 3)$

Learners should have drawn a coordinate grid to help explain their answers.

4 **a** *P* is $(1, 5)$ *Q* is $(5, 1)$ The midpoint of *PQ* is $(3, 3)$

b *R* is $(5, -2)$ *S* is $(1, -6)$ The midpoint of *RS* is $(3, -4)$

c *T* is $(-6, -4)$ *U* is $(-2, -2)$ The midpoint of *TU* is $(-4, -3)$

d *V* is $(-6, 4)$ *W* is $(-2, 2)$ The midpoint of *VW* is $(-4, 3)$

5 **a** $\left(\frac{1+7}{2}, \frac{4+6}{2}\right) = \left(\frac{8}{2}, \frac{10}{2}\right) = (4, 5)$

b $\left(\frac{18+8}{2}, \frac{0+8}{2}\right) = \left(\frac{26}{2}, \frac{8}{2}\right) = (13, 4)$

c $\left(\frac{7+5}{2}, \frac{3+10}{2}\right) = \left(\frac{12}{2}, \frac{13}{2}\right) = \left(6, 6\frac{1}{2}\right)$

d $\left(\frac{1+4}{2}, \frac{4+15}{2}\right) = \left(\frac{5}{2}, \frac{19}{2}\right) = \left(2\frac{1}{2}, 9\frac{1}{2}\right)$

6 **a** **i** $(-1, 0)$ **ii** $(-2, 5)$

iii $(5, 5)$

b learners' diagrams and checks

7 **a** $(7, 7)$ **b** $(1, -5)$ **c** $(1, 1)$

8 Sasha is incorrect. Learners' explanations.

Example: She has added the *x*-coordinate of one point to the *y*-coordinate of the other point.

Correct answer should be:

x-coordinate: $\frac{-3+5}{2} = \frac{2}{2} = 1$

y-coordinate: $\frac{8+-2}{2} = \frac{6}{2} = 3$

Midpoint is at $(1, 3)$

9 **a** $(0.5, -0.5)$

b The midpoint is $\left(\frac{4+-3}{2}, \frac{-1+0}{2}\right) =$ $(0.5, -0.5)$ which is the midpoint of *AC*.

10 **a** $(3.0, 4.3)$

b $(-0.7, 2.4)$

11 $(-2.5, -3.5)$ Learners' explanations. Example: Found the midpoint of *AC* as this is the diagonal and the centre of a square is at the midpoint of the diagonals.

12 *Q* $(10, 5)$

13 *C* is the midpoint of $AB = \left(\frac{16+0}{2}, \frac{0+8}{2}\right) = (8, 4)$

D is the midpoint of $BC = \left(\frac{0+8}{2}, \frac{8+4}{2}\right) = (4, 6)$

E is the midpoint of $CD = \left(\frac{8+4}{2}, \frac{4+6}{2}\right) = (6, 5)$

14 Any three pairs of possible coordinates for *G* and *H*, so that the midpoint is at $(3, 2)$.

Examples: $(0, 0)$ and $(6, 4)$, $(1, 1)$ and $(5, 3)$, $(2, 0)$ and $(4, 4)$

Exercise 14.3

1 **A** and **iv**, **B** and **vi**, **C** and **i**, **D** and **v**, **E** and **iii**, **F** and **ii**

2 **a** The column vector $\begin{pmatrix} 4 \\ 5 \end{pmatrix}$ means move the shape 4 units right and 5 units up.

b The column vector $\begin{pmatrix} -1 \\ 6 \end{pmatrix}$ means move the shape 1 unit left and 6 units up.

c The column vector $\begin{pmatrix} 2 \\ -4 \end{pmatrix}$ means move the shape 2 units right and 4 units down.

3 a **b**

c

4
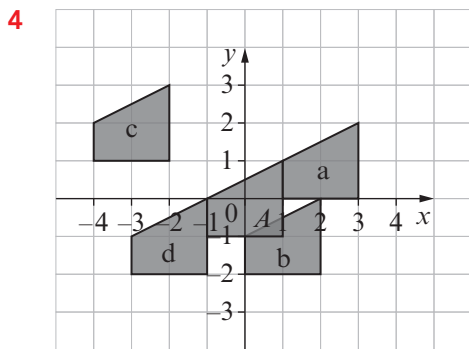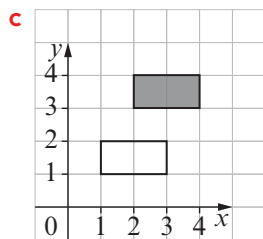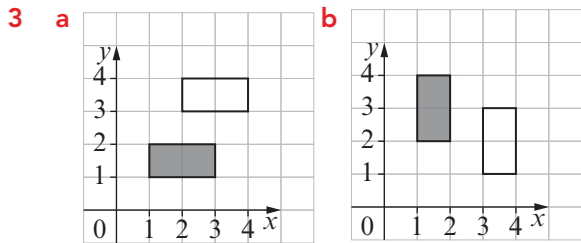
5 Adah is incorrect. Learners' explanations. Example: She has translated shape B using the column vector $\begin{pmatrix} 3 \\ 2 \end{pmatrix}$ not $\begin{pmatrix} 2 \\ 3 \end{pmatrix}$.

6 a, b
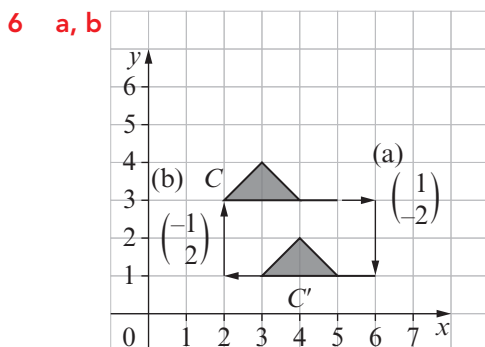
c learners' explanations

Example: Change the signs on the numbers in the original column vector.

7 a $\begin{pmatrix} -3 \\ -4 \end{pmatrix}$ **b** $\begin{pmatrix} -7 \\ 8 \end{pmatrix}$

c $\begin{pmatrix} 1 \\ 9 \end{pmatrix}$ **d** $\begin{pmatrix} -m \\ -n \end{pmatrix}$

8 a $\begin{pmatrix} 6 \\ 2 \end{pmatrix}$ **b** $\begin{pmatrix} 3 \\ -3 \end{pmatrix}$

c $\begin{pmatrix} -9 \\ 1 \end{pmatrix}$ **d** $\begin{pmatrix} -3 \\ -5 \end{pmatrix}$

9 a $H'(-4, 4)$, $I'(-3, 4)$ and $J'(-1, 2)$

b
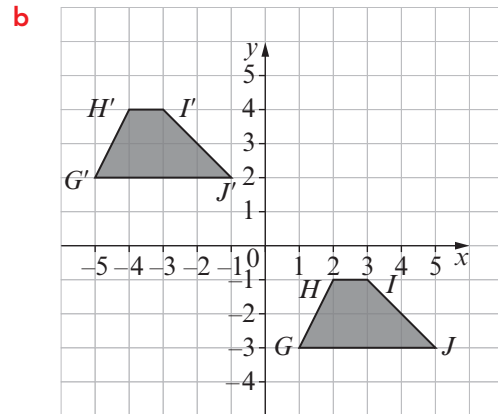
c learners' checks

10 a Aki is not correct; learners' explanations

Example: Coordinates of P' are

$(-1, 1) + \begin{pmatrix} 6 \\ -4 \end{pmatrix} = (-1 + 6, 1 + -4) = (5, -3)$

not $(5, -5)$

b $Q'(6, -1)$, $R'(9, -1)$ and $S'(8, -3)$

c
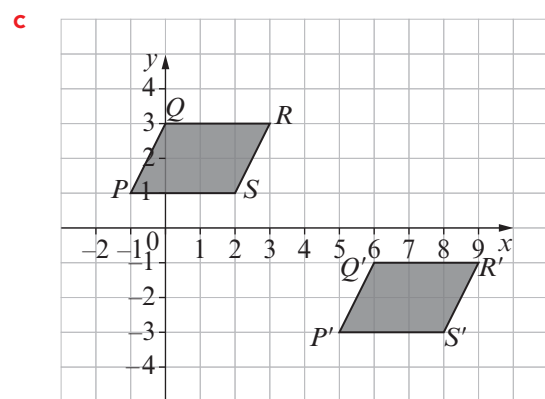
11 a $\begin{pmatrix} 3 \\ 1 \end{pmatrix}$

b $S'(5, -1)$, $T'(1, -1)$ and $U'(1, 3)$

c

b

12 a, b
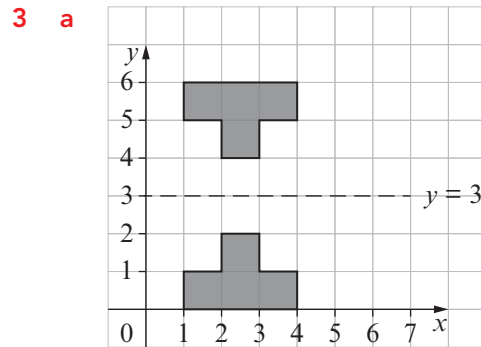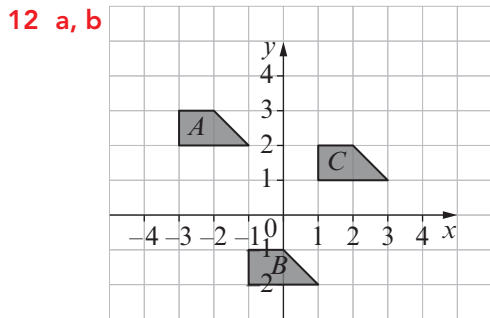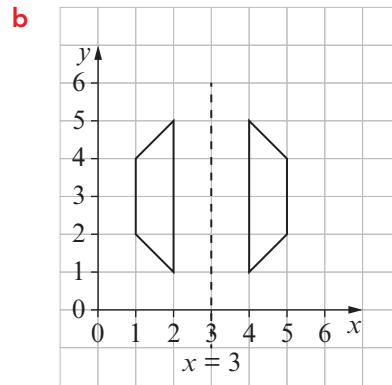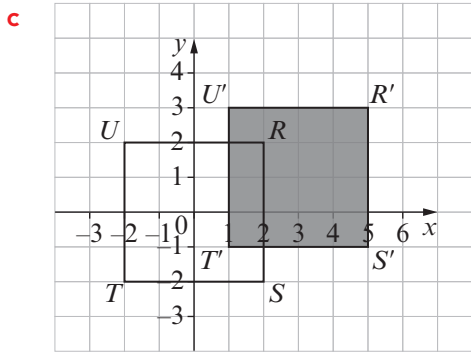
c $\begin{pmatrix} 4 \\ -1 \end{pmatrix}$ Add them together and you get the answer $\begin{pmatrix} 2 \\ -4 \end{pmatrix} + \begin{pmatrix} 2 \\ 3 \end{pmatrix} = \begin{pmatrix} 4 \\ -1 \end{pmatrix}$

d i $\begin{pmatrix} 6 \\ 5 \end{pmatrix}$ **ii** $\begin{pmatrix} -5 \\ 2 \end{pmatrix}$

e $\begin{pmatrix} a+c \\ b+d \end{pmatrix}$
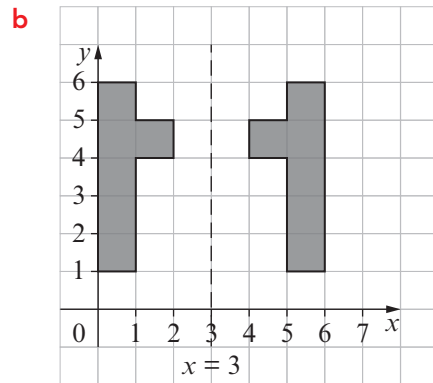
Exercise 14.4

1 **A** is $y = 5$, **B** is $y = 3$, **C** is $y = 1$, **D** is $x = 2$, **E** is $x = 5$

2 a

3 a

b

c

4 a

x = 4

b

y = 4

c

y = 3
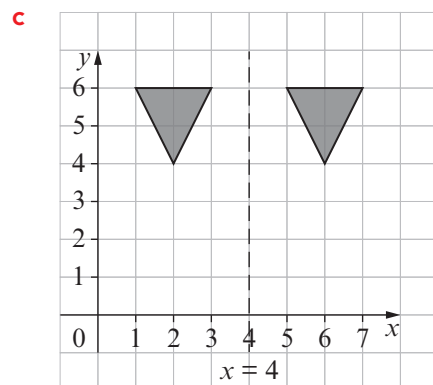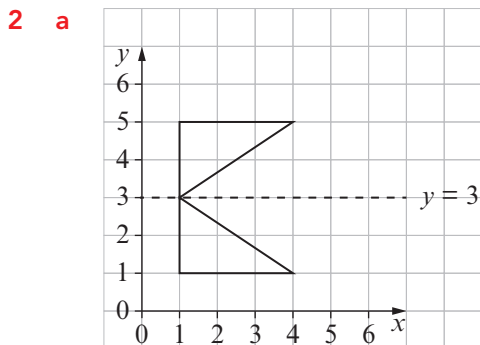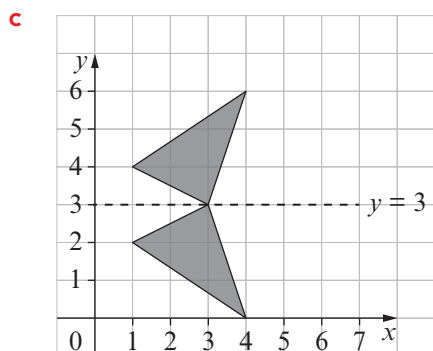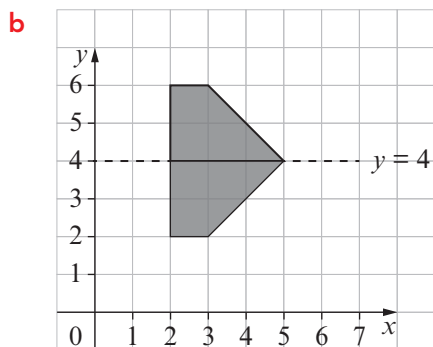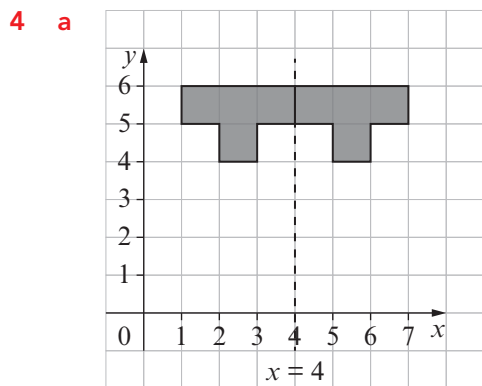
5 a learners' explanations

Example: She has *A′* in the correct position, but she has copied the shape and not reflected it.

b

x = −2

6

7 a

x = 4

b

y = 3
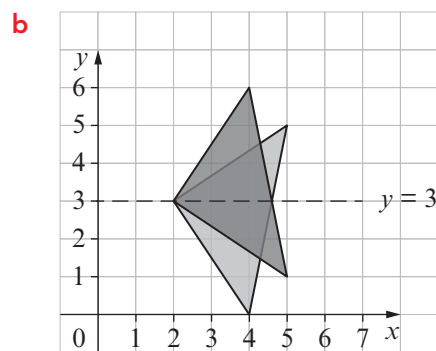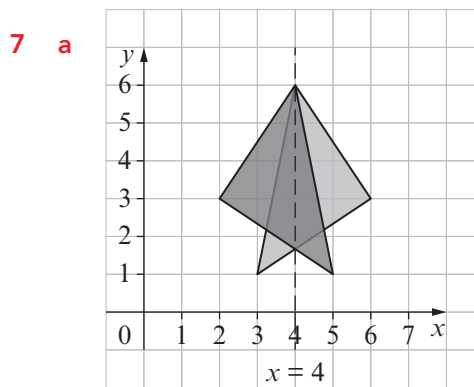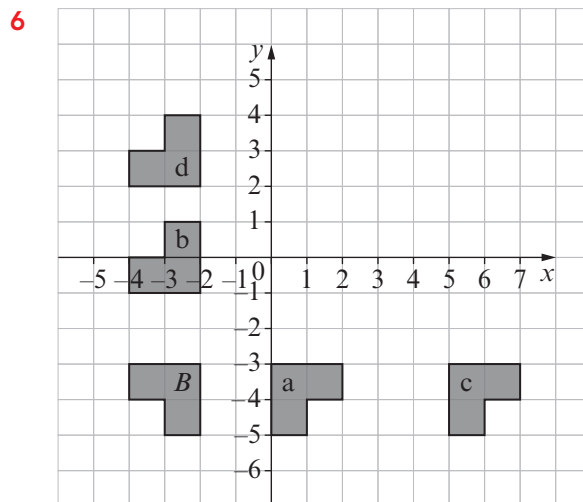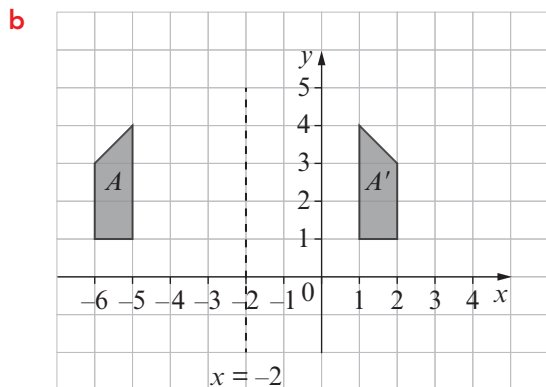
c learners' explanations

Example: Reflect one vertex at a time in the mirror line.

8
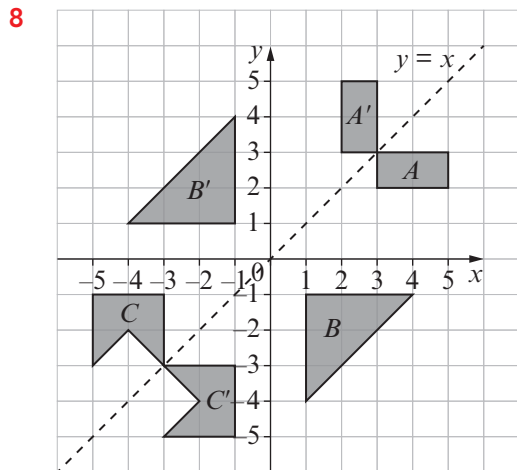
9 a

Object (A)	(3, 3)	(3, 2)	(5, 2)	(5, 3)
Image (A')	(3, 3)	(2, 3)	(2, 5)	(3, 5)

b learners' explanations

Example: The x and y coordinates are swapped over.

c learners' rule

Example: For each coordinate (a, b) on the object, the corresponding coordinate on the image is (b, a).

10 a A (2, 6), B (7, 6), C (6, 3) and D (0, 2)

b A' (6, 2), B' (6, 7), C' (3, 6) and D' (2, 0)

c, d

11 a
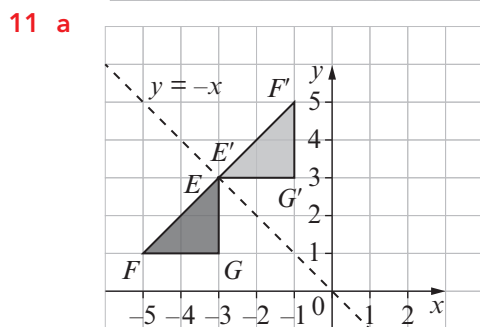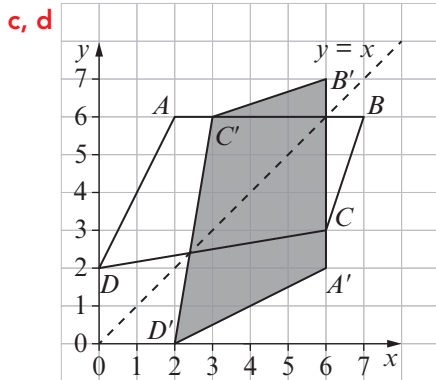
b

Object	E (−3, 3)	F (−5, 1)	G (−3, 1)
Image	E (−3, 3)	F' (−1, 5)	G' (−1, 3)

c learners' explanations

Example: The x and y coordinates are swapped over and the signs are changed.

d learners' rule

Example: For each coordinate (a, b) on the object, the corresponding coordinate on the image is $(-b, -a)$.

12 a P (3, −5), Q (1, −2), R (0, −3) and S (1, −5).

b P' (5, −3), Q' (2, −1), R' (3, 0) and S' (5, −1).

c, d
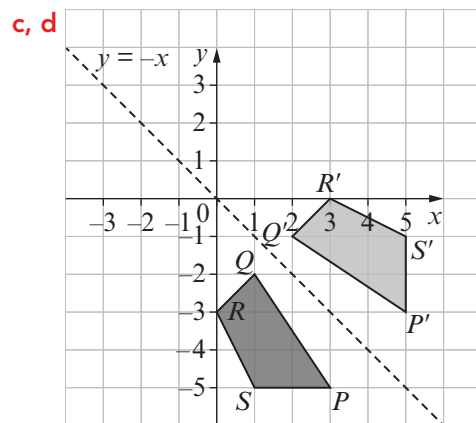
13 a iii $x = 4$ **b** i $x = -1$

c i $y = 2.5$ **d** iii $y = 0$

e ii $x = -0.5$

14 Arun is incorrect; learners' explanations.

Example: He needs to translate shape G using the column vector $\begin{pmatrix} 0 \\ 5 \end{pmatrix}$ not $\begin{pmatrix} -1 \\ 5 \end{pmatrix}$ for the image to be in the same position as shape A.

Exercise 14.5

1 **a**

b

2 **a**

b

3 **a**

b

4 **a**

b

c

5
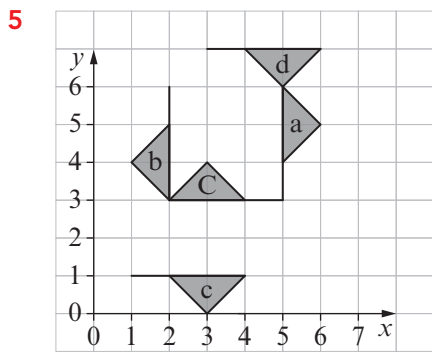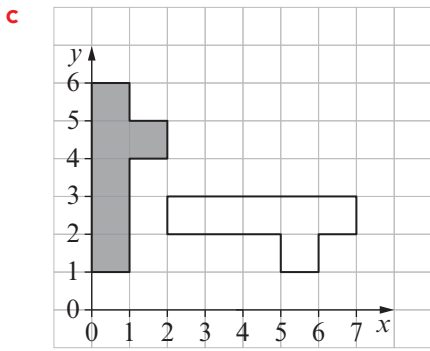
6 a Learners' explanations. Example: He has rotated it about the centre (2, 3) not (3, 2).

b
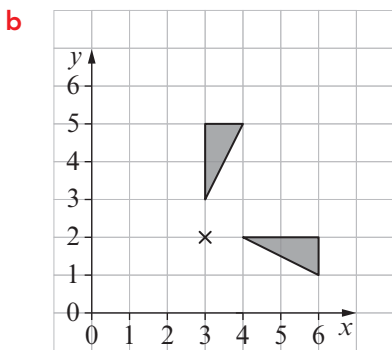
7 a Rotation 180º, centre (4, 2)

b Rotation 180º, centre (−1, 2)

8 a vi **b** iii **c** ii

d i **e** iv **f** v

9 a Rotation 90° clockwise, centre (−5, −1)

b Rotation 180°, centre (1, −2)

c Rotation 90° anticlockwise, centre (5, −3)

d Rotation 180°, centre (4.5, −1)

e Rotation 180°, centre (3, 2)

10 a–c
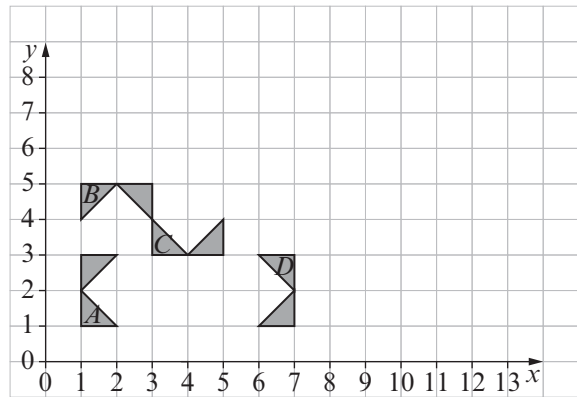
d Rotation 180°, centre (4, 2)

e Reflection in line $x = 4$

11 a
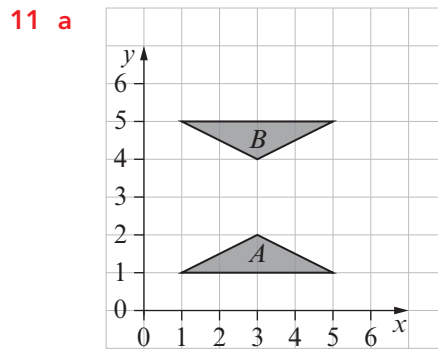
b Transformation 1: rotation of 180° about centre (3, 3); Transformation 2: reflection in the line $y = 3$

12 a–c
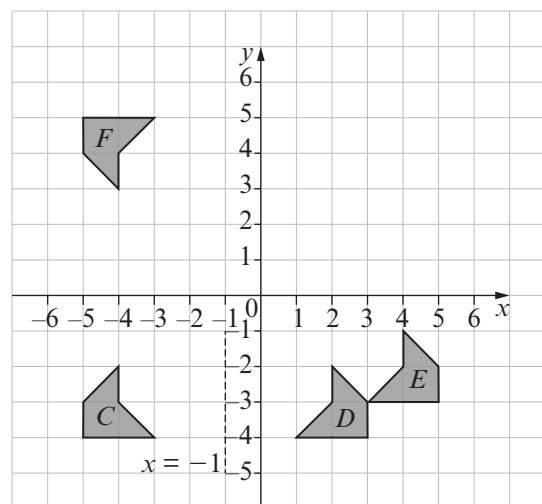
d Reflection in the line $y = 0.5$

e Rotation 180°, centre (−1, 0.5)

Exercise 14.6

1 a

2 × 2 = 4 squares
C
2 squares

b

C
1 × 2 = 2 } 1 square

c

C 2
1 1
2

2 a

C

b

C

c

C

d

C
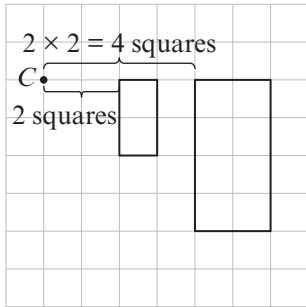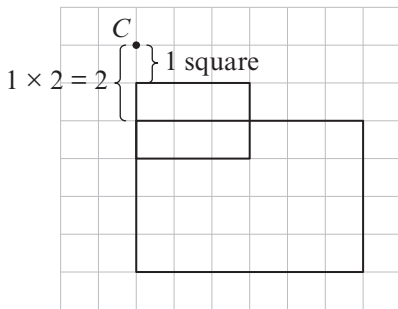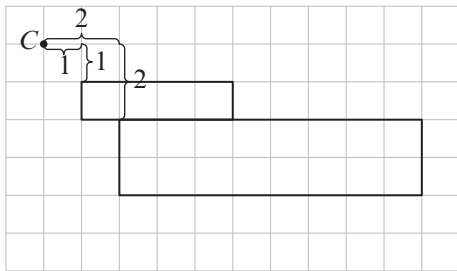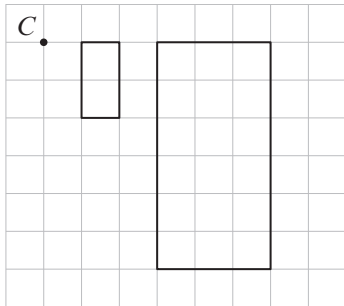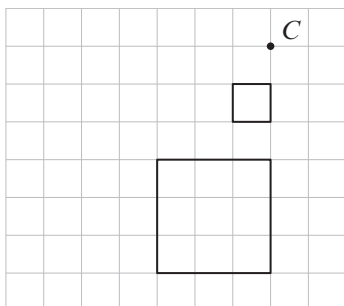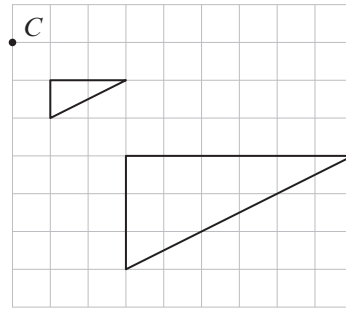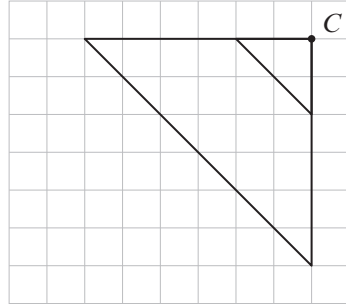
3
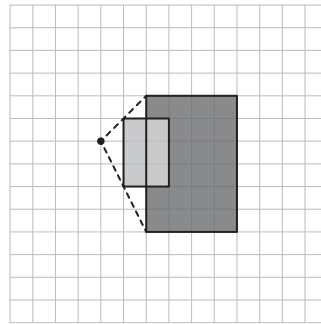
scale factor 2
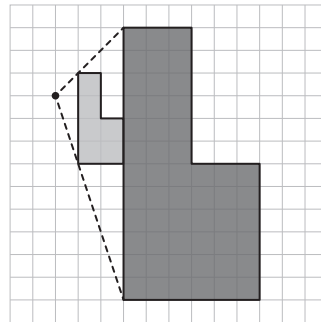
scale factor 3
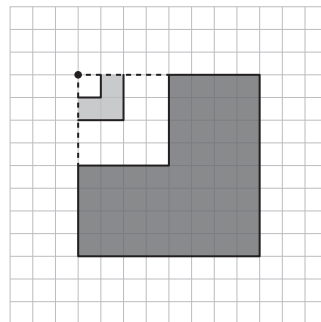
scale factor 4

scale factor 2
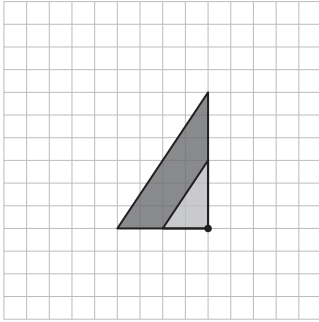
scale factor 3
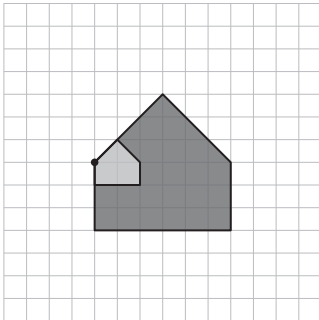
scale factor 2
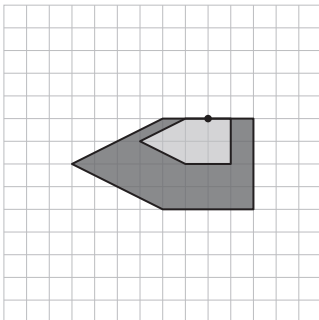
4 a Learners' explanations. Example: The top and bottom sides of the trapezium have not been enlarged by a scale factor of three.

b
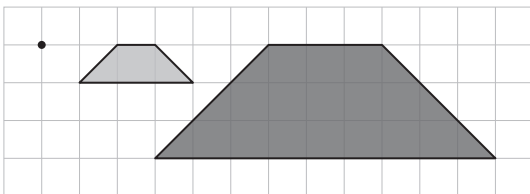
5 a
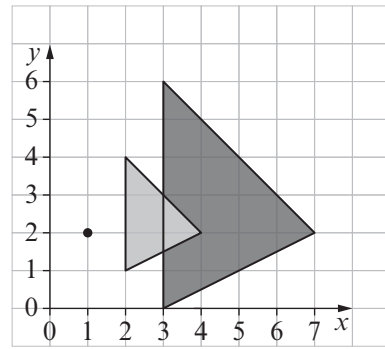
b (3, 0), (3, 6), (7, 2)

c There is not an invariant point on the object and image.

Learners' explanations. Example: The centre of enlargement is outside the shape, so there are no points that are in the same place on the object and the image.

6 a
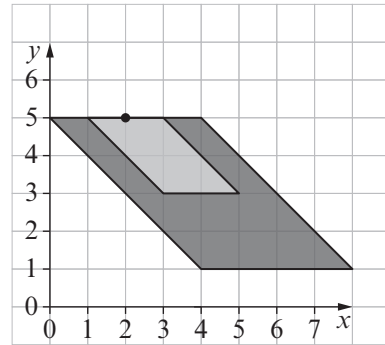
b (0, 5), (4, 1), (8, 1), (4, 5)

c There is an invariant point on the object and image, at (2, 5).

Learners' explanations. Example: The centre of enlargement (2, 5) is on the edge of the shape, so the centre of enlargement is in the same place on the object and the image.

7 *A* scale factor 2, centre (1, 12), *B* scale factor 3, centre (23, 14), *C* scale factor 3, centre (24, 3), *D* scale factor 3, centre (7.5, 10)

8 a scale factor 2

b Sofia is correct. Learners' explanations. Example: I tried both centres of enlargement and Sofia's worked but Zara's didn't.

9 **a** Yes, scale factor 3.
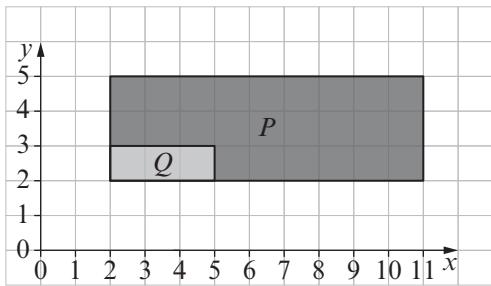
b (2, 2)

Exercise 15.1

1

Number of miles	5	10	15	20	25	30	35	40
Number of kilometres	8	16	24	32	40	48	56	64

2 **a** T   **b** F   **c** F

   d T   **e** F

3 Sofia is incorrect. Her mother has to travel further as 18 miles is further than 18 km.

4 **a** 16 km $16 \div 8 = 2$ $2 \times 5 = 10$ miles

   b 48 km $48 \div 8 = 6$ $6 \times 5 = 30$ miles

   c 72 km $72 \div 8 = 9$ $9 \times 5 = 45$ miles

5 **a** 15 miles $15 \div 5 = 3$ $3 \times 8 = 24$ km

   b 25 miles $25 \div 5 = 5$ $5 \times 8 = 40$ km

   c 40 miles $40 \div 5 = 8$ $8 \times 8 = 64$ km

6 **a** 55 miles   **b** 45 miles

   c 75 miles   **d** 125 miles

7 **a** 48 km   **b** 480 km

   c 72 km   **d** 7200 km

8 128 km is further (75 miles = 120 km)

9 296 km is further (296 km = 185 miles)

10 **a** 104 km = 65 miles

   b 95 miles = 152 km

   c 190 miles = 304 km (or 168 miles = 105 km)

   d 105 km = 168 miles (or 304 km = 190 miles)

11 **a** $11\frac{1}{5}$ km or 11.2 km

   b $33\frac{3}{5}$ km or 33.6 km

   c $62\frac{2}{5}$ km or 62.4 km

12 **a** $12\frac{1}{2}$ miles   **b** $21\frac{1}{4}$ miles

   c $39\frac{3}{8}$ miles

13 Johannes is incorrect. He should get about $11 000 for his car, not $10 500.

   $045\,605 - 008\,935 = 36\,670$ miles

   $36\,670 \div 5 \times 8 = 58\,672$ km

   $58\,672 \times 0.05 = \$2933.6$

   $13\,995 - 2933.6 = \$11\,061.40$

14 688 km

15 **a** **i** 15   **ii** 20

   b **i** 24   **ii** 32

   c missing numbers are: 750, 750, 1500

   d 1000

   e **i** 450   **ii** 300

   f 1 hour : 5 km   1 hour : 600 m

   60 minutes : 5 km   60 minutes : 600 m

   12 minutes : 1 km   1 minute : 10 m

   g **i** 5 hours 33 minutes

   ii 6 hours 54 minutes

Exercise 15.2

1 **a** Area = base × height = $4 \times 2 = 8$ cm²

   b Area = base × height = $2 \times 3 = 6$ cm²

   c Area = base × height = $3 \times 3 = 9$ cm²

2 **a** 160 mm²   **b** 45 m²

3

	a	b	c
Step 1: $4 + 6 = 10$	$5 + 7 = 12$	$8 + 12 = 20$
Step 2: $10 \div 2 = 5$	$12 \div 2 = 6$	$20 \div 2 = 10$
Step 3: $5 \times 3 = 15$ cm²	$6 \times 6 = 36$ cm²	$10 \times 9 = 90$ cm²

4 **a** 42 m²   **b** 60 cm²

5 **a** learners' explanations

Example: Jen has used mixed units. The length is in cm and the height is in mm. She needs to have both dimensions in the same units.

b 9.6 cm² or 960 mm²

6 **a** 24 m²   **b** 115 mm²

c 52.5 cm²

7 **a** 72 cm²   **b** 73.16 cm²

8 **a, b** $A = v$ (6×4)

$B = ii$ $(\frac{1}{2} \times 10 \times 3)$

$C = iv$ (5×4)

$D = i$ $(\frac{1}{2} \times (4+6) \times 3)$

c card **iii**

9 155 mm or 15.5 cm

10 **a** 119 cm²   **b** 57.5 m²

11 6.3 cm or 63 mm

12 Kai is incorrect. The shaded area is 885 cm² not 875 cm².

Area of rectangle $= 30 \times 40 = 1200$ cm²

Area of trapezium $= \frac{1}{2}(14 + 28) \times 15 = 315$ cm²

Shaded area $= 1200 - 315 = 885$ cm²

13 **a** $\frac{1}{6}$ m²   **b** $\frac{5}{14}$ m²   **c** $\frac{19}{60}$ m²

14 **a** 5120 square kilometres

b 2000 square miles

15 $300

Exercise 15.3

1 **a** Area of cross-section $= \frac{1}{2} \times b \times h$

$= \frac{1}{2} \times 3 \times 4 = 6$ cm²

Volume $=$ area of cross-section $\times$ length

$= 6 \times 8$

$= 48$ cm³

b Area of cross-section $= \frac{1}{2} \times b \times h$

$= \frac{1}{2} \times 5 \times 6 = 15$ m²

Volume $=$ area of cross-section $\times$ length

$= 15 \times 9$

$= 135$ m³

2 **a** 800 cm³   **b** 90 m³

3 learners' explanations

Example: When Anil has worked out the area of the cross-section he hasn't used the perpendicular height of the triangle. He has used the length of the 9 cm side instead of the perpendicular height of 8 cm.

Correct answer is 1200 cm³

4 **a** Joe

Area of cross-section $= \frac{1}{2} \times b \times h$

$= \frac{1}{2} \times 8 \times 5$

$= 20$ m²

Volume $=$ area of cross-section $\times$ length

$= 20 \times 6$

$= 120$ m³

Alice

Volume $=$ area of cross section $\times$ length

$= \frac{1}{2} \times b \times h \times l$

$= \frac{1}{2} \times 8 \times 5 \times 6$

$= 120$ m³

b learners' answers

5

	Base	Height	Length	Volume
a	6 cm	10 cm	20 mm	60 cm³
b	0.5 cm	12 mm	6 mm	180 mm³
c	1.5 m	6 m	80 cm	3.6 m³
d	40 mm	4 cm	400 mm	320 cm³

6 **a** 500 cm³   **b** 9600 mm³

7 $V = 72$ cm³, $A = 18$ cm², $l = 4$ cm

$V = 84$ cm³, $A = 12$ cm², $l = 7$ cm

$V = 90$ cm³, $A = 15$ cm², $l = 6$ cm

$V = 108$ cm³, $A = 9$ cm², $l = 12$ cm

8 25 mm²

9 **a** 32 cm²

b Any two numbers that multiply to give 64. Examples:

base $= 8$ cm and height $= 8$ cm

base $= 16$ cm and height $= 4$ cm

learners' explanations

Example: I divided the volume by the length to give me an area of the triangle of $32\,cm^2$. $\frac{1}{2}\times b\times h=32$, so $b\times h=64$, so I can choose any two numbers that multiply to give 64.

10 $7\,mm$

11 a $336\,m^3$

b any two examples such that $\frac{1}{2}\times b\times h\times l=336\,m^3$

Examples: ① $b=16\,m$, $h=6\,m$, $l=7\,m$
② $b=8\,m$, $h=6\,m$, $l=14\,m$

12 Hari is incorrect.

The mass of the ramp is $1680\,kg$ which is less than $1700\,kg$, not more than $1700\,kg$.

Volume of ramp $=\frac{1}{2}\times5\times0.2\times1.4=0.7\,m^3$

Mass of ramp $=0.7\times2400=1680\,kg$

Exercise 15.4

1 a Area A $=10\times9=90\,cm^2$

Area B $=12\times9=108\,cm^2$

Area C $=$ Area A $=90\,cm^2$

Area D $=\frac{1}{2}\times12\times8=48\,cm^2$

Area E $=$ Area D $=48\,cm^2$

Total area $=90+108+90+48+48=384\,cm^2$

b Area A $=5\times6=30\,cm^2$

Area B $=4\times6=24\,cm^2$

Area C $=3\times6=18\,cm^2$

Area D $=\frac{1}{2}\times4\times3=6\,cm^2$

Area E $=$ Area D $=6\,cm^2$

Total area $=30+24+18+6+6=84\,cm^2$

c Area A $=8\times8=64\,cm^2$

Area B $=\frac{1}{2}\times8\times10=40\,cm^2$

Area of all four triangles $=4\times40=160\,cm^2$

Total area $=64+160=224\,cm^2$

2 a learners' explanations

Example: He has worked out the volume not the surface area.

b $840\,cm^2$

3 a i
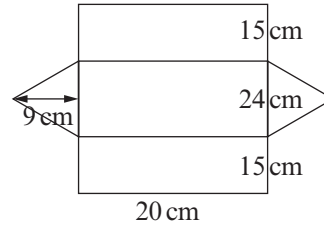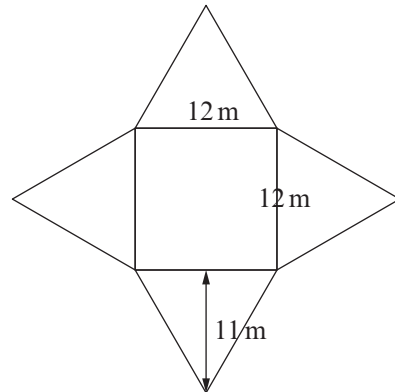

ii $1296\,cm^2$

b i
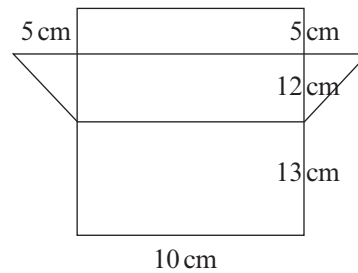
ii $360\,cm^2$

c i

ii $408\,m^2$

d i
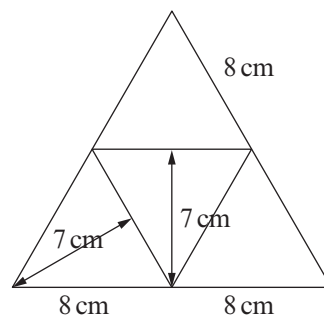
ii $112\,cm^2$

4 Yes. Mia is correct.

Surface area of the triangular prism $= 2 \times \frac{1}{2} \times 8 \times 3 + 8 \times 2.75 + 2 \times 5 \times 2.75 = 73.5\,cm^2$

Surface area of the cube $= 6 \times 3.5 \times 3.5 = 73.5\,cm^2$

5 Surface area of pyramid $= \left(\frac{1}{2} \times 7 \times 9\right) \times 4 = 126\,m^3$

Surface area of cuboid $= 2 \times (3 \times 4) + 2 \times (3 \times 2.5) + 2 \times (4 \times 2.5) = 59\,m^3$

$2 \times$ Surface area of cuboid $= 2 \times 59 = 118\,m^3$

$126\,m^3 > 118\,m^3$ so the surface area of the triangular-based pyramid is more than double the surface area of the cuboid.

6 Razi is correct; the triangular prism has the smaller surface area.

Surface area of prism $\approx 2 \times (\frac{1}{2} \times 90 \times 80) + (90 \times 8) + (80 \times 8) + (100 \times 8) = 9360\,cm^2$

Surface area of cube $\approx 6 \times 40 \times 40 = 9600\,cm^2$

7 a Surface area $= 22x^2\,cm^2$

b $748\,cm^2$

8 height $= 16\,mm$

Exercise 16.1

1 a 12

b i 5

ii 2

c 3

d 30

2

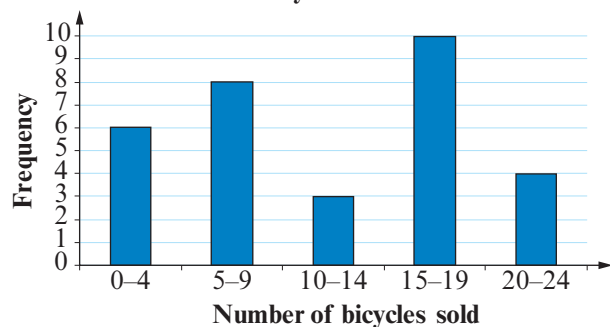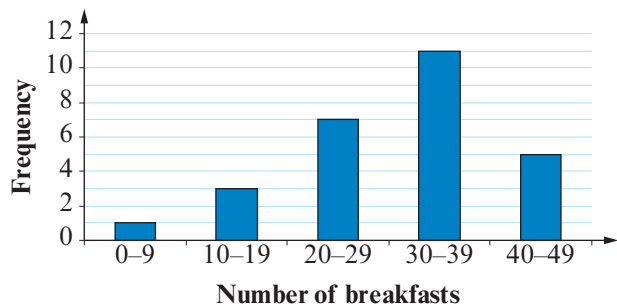**Number of bicycles sold in one month**

3 a

Number of breakfasts sold each day in a cafe during one month

b 3

c 23; added up the frequencies for 20−29, 30−39 and 40−49

d 27

e There is not a month that has 27 days.

f The manager is not correct; learners' explanations

Example: You can only tell that the greatest number of breakfasts sold was between 40 and 49. You cannot tell the exact value.
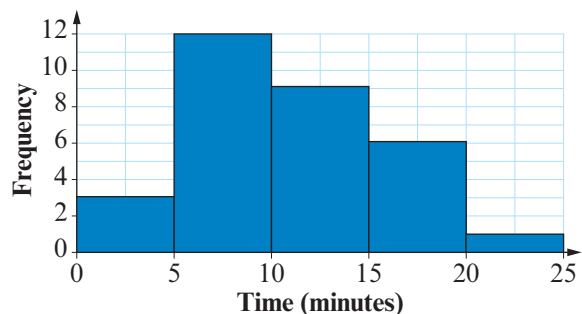
4 a learners' explanations

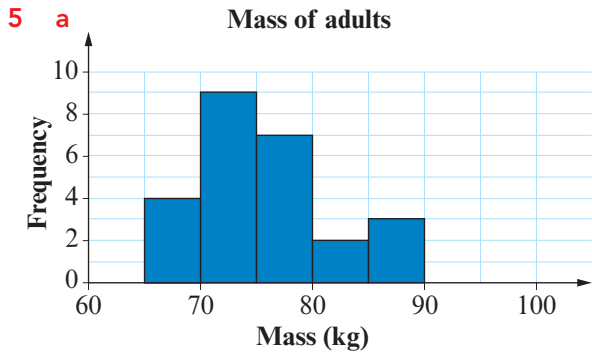Example: A time, in minutes, that is greater than zero minutes, but less than or equal to 5 minutes.

b learners' explanations

Example: Someone might take 4.5 minutes or 9 minutes 15 seconds, and so if you used the classes 0−4, 5−9, etc. you wouldn't be able to record their time.

c $10 < t \leqslant 15$

d

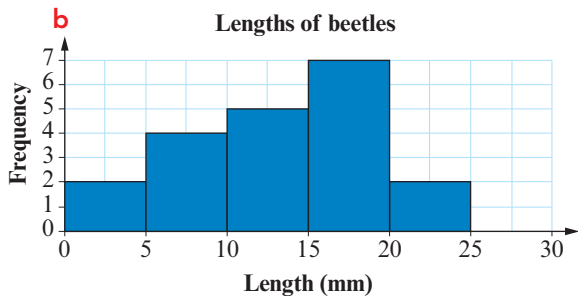**Time taken to complete a puzzle**

5 a

Mass of adults

b Zara is incorrect. Learners' explanations. Example: You can only tell that the heaviest adult weighed more than 85 kg less than or equal to 90 kg. You cannot tell the exact value.
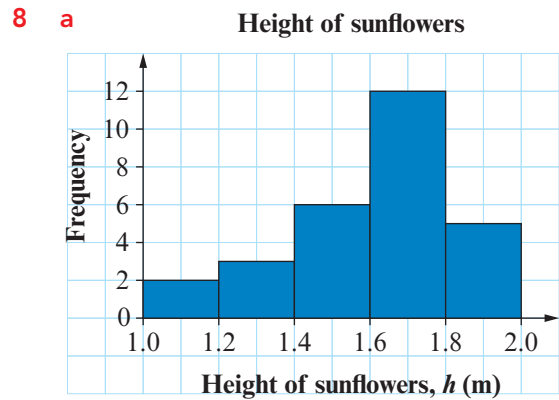
c 5

6 a

Length, *l* mm	Frequency
$0 \leqslant l < 5$	2
$5 \leqslant l < 10$	4
$10 \leqslant l < 15$	5
$15 \leqslant l < 20$	7
$20 \leqslant l < 25$	2

b

Lengths of beetles

c 14; learners' explanations

Example: Added up the frequencies for the last three groups as these all have a length greater than or equal to 10 mm.

7

Height, *h* cm	Frequency
$0 < h \leqslant 10$	3
$10 < h \leqslant 20$	6
$20 < h \leqslant 30$	4
$30 < h \leqslant 40$	2

8 a

Height of sunflowers

b 17

c No. It could not be exactly 1 m as '1.0<' means that the height must be greater than 1.0 m.

d Not really. It could be 2 m, but you can't tell from grouped data information; the tallest sunflower could be anywhere from just above 1.8 m to 2 m.
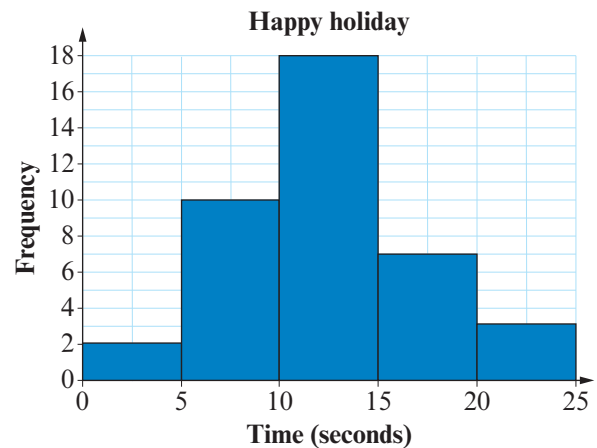
e 28

9 a learners' own frequency tables with their chosen class intervals

Example:

Time, *t* seconds	Frequency
$0 < h \leqslant 5$	2
$5 < h \leqslant 10$	10
$10 < h \leqslant 15$	18
$15 < h \leqslant 20$	7
$20 < h \leqslant 25$	3

b learners' own frequency diagrams

Example:

Happy holiday

Exercise 16.2

1 a i 5 million **ii** 6.5 million

    b 2007   **c** 2006 and 2007

    d 2010 and 2011   **e** 2008 to 2009

    f There was a large increase in production from 2006 to 2007, then a small but fairly steady increase in production from 2007 to 2010; from 2010 to 2011 there was no increase in production.

2 a i 65   **ii** 0

    b February

    c November and December

    d Ski rental starts off high for the first three months of the year (65, 70 and 65), then goes down to zero over the next three months. Kelly does not rent out any skis from June to September, but rentals improve from October increasing rapidly to reach 55 in December.

3 a $4700

    b i 2008 and 2009

       ii 2018 and 2019

       iii 2013 and 2014

    c The overall trend is up. Although she has had years when her investment has gone down, overall her investment is increasing in value.

4 a Number of skateboards sold by a shop over one year
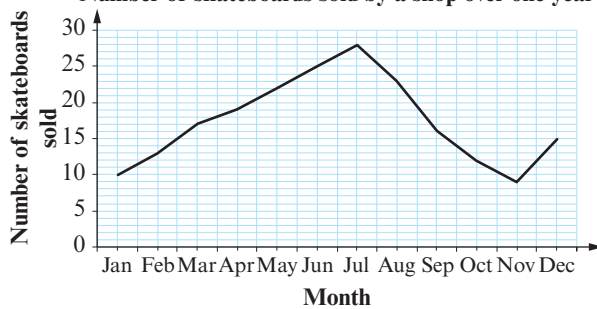
    b i November and December

       ii August and September

    c From January to July, sales of skateboards increase. From July to November the sales then decrease each month, before a final increase in sales in December.

5 a Average price of books sold from Brendan's Books
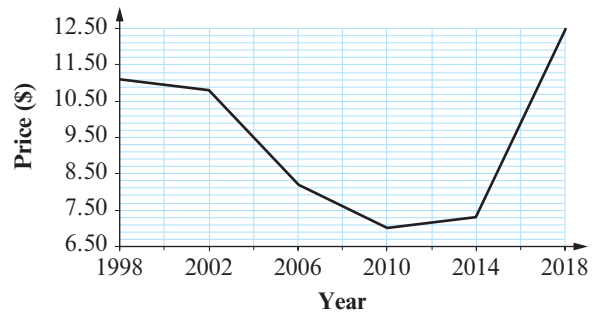
    b 2014 and 2018

    c $9.50

    d From 1998 to 2010 there was a decline in the average price of books sold. After 2010 the average price started to increase gradually, with a sharp increase from 2014 to 2018.

6 a i Overall Freetown's points total increased, showing that they were improving year on year.

       ii Overall Newtown's points total decreased, showing that they were getting worse year on year.

    b learners' answers and explanations Example: No, they need to improve by another 5 points, which is more than they have been improving for the last few years.

    c learners' answers and explanations Example: Yes, they only need to go down another 2 points, which is quite likely looking at the last few years' scores.

7 a Number of visitors to a theme park

b Learners' description. Example: The number of visitors increases from Spring to Summer and then from Summer to Autumn. The number then decreases from Autumn to Winter and from Winter to the following Spring.

c Yes. Learners' explanation. Example: In 2019 and 2020 the numbers increase from Spring to Summer to Autumn, then decrease from Autumn to Winter. This also happens in the parts of 2018 and 2021 that we have data for.

d Learners' description. Example: Overall the number of visitors each year is decreasing.

e Learners' prediction. Example: 290 000 visitors. (accept answer in the range 275 000 to 298 000)

f Learners' explanation. Example: It is a prediction into the future, so it may not actually happen.

8 a Visitor numbers to a riding stables per month
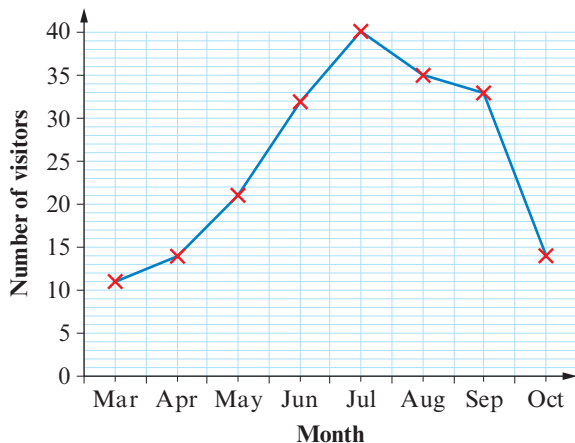
b From March to July the visitor numbers increase, then from July to October the numbers decrease. There is a sharp drop in numbers from September to October.

9 a **Mean monthly temperature in Kangerlussuaq and Port Stanley**
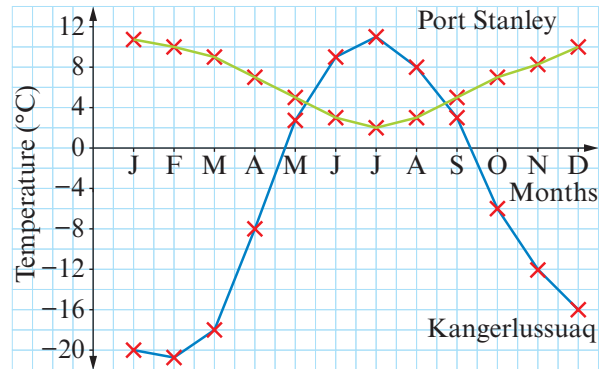
b i At the start and end of the year the temperatures are very cold (−20 °C and −16 °C). Temperatures increase rapidly from March to June, reaching a maximum of 11 °C in July, before decreasing again.

ii At the start and end of the year, the temperatures are 11 °C and 10 °C. The temperatures decrease gradually to a minimum of 2 °C in July before increasing gradually again.

c learners' work

Exercise 16.3

1 30, 38, 39, 39, 42, 44, 46, 47, 47, 48

2 a 16 °C, 17 °C, 18 °C, 19 °C, 20 °C, 22 °C, 22 °C

  b i 22 °C   **ii** 19 °C   **iii** 6 °C

3 a 18, 19, 19, 20, 21, 22, 25, 26, 29, 32, 35

  b i 19   **ii** 22   **iii** 17

4 a 19   **b** 23 minutes   **c** 14

  d i 35 minutes   **ii** 37 minutes

  iii 26 minutes

5 a 22   **b** 105 minutes

  c 10   **d** 11

  e Because one film lasts exactly 2 hours.

  f i 113 minutes   **ii** 121 minutes

  iii 32 minutes

6 a Key: 5 | 8 means 5.8 g

5	8 9 9
6	0 1 2 4 4 4 9 9
7	1 2 3 3 5 8
8	0 2 5 6 9
9	0 2 5

b 17   **c** $\frac{2}{25}$ **d** 44%

e i 6.4 g   **ii** 7.2 g   **iii** 3.7 g

7 a Key: 16 | 2 means 162 kg

16	2 5 6
17	0 2 3 5 5 5 6 9
18	0 0 8 9
19	0 4 8 9
20	0 7 8 8 9

b 4   **c** $\frac{11}{24}$ **d** 37.5%

e i 175 kg   **ii** 180 kg   **iii** 185 kg

f learners' choices of median or mean with reason

Example: The mean because it uses all the values and sits nicely in the middle of the data. There are no extreme values to affect the mean.

g Tia is incorrect. The range is 47 kg not 45 kg because $209 - 162 = 47$ kg

8 a

	Least distance	Greatest distance	Range	Median distance	Mean distance	Modal distance
Right hand	3	38	35	20	22	16
Left hand	2	38	36	15	16	8

b The throws with the right hand had a larger mean distance than the throws with the left hand. The greatest distance thrown was the same for both hands.

c Four, because four students threw under 10 m with their right hand and threw 28 or more with their left hand.

9 **Emails received by employees in one day**

Legend: 0–9, 10–19, 20–29, 30–39, 40–49
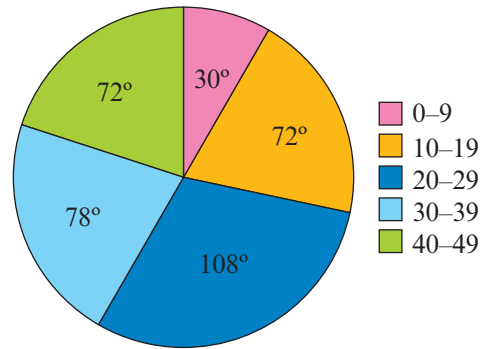
Exercise 16.4

1 a 40°

b Giraffe: $\frac{120}{360} = \frac{1}{3}$ Zebra: $\frac{40}{360} = \frac{1}{9}$

Elephant: $\frac{160}{360} = \frac{4}{9}$ Impala: $\frac{40}{360} = \frac{1}{9}$

c Giraffe: $\frac{1}{3} \times 45 = 45 \div 3 = 15$

Zebra: $\frac{1}{9} \times 45 = 45 \div 9 = 5$

Elephant: $\frac{4}{9} \times 45 = 45 \div 9 \times 4 = 20$

Impala: $\frac{1}{9} \times 45 = 45 \div 9 = 5$

2 a 90°

b Dolphin: $\frac{130}{360} = \frac{13}{36}$ Shark: $\frac{30}{360} = \frac{1}{12}$

Turtle: $\frac{110}{360} = \frac{11}{36}$ Whale: $\frac{90}{360} = \frac{1}{4}$

c Dolphin: $\frac{13}{36} \times 72 = 72 \div 36 \times 13 = 26$

Shark: $\frac{1}{12} \times 72 = 72 \div 12 = 6$

Turtle: $\frac{11}{36} \times 72 = 72 \div 36 \times 11 = 22$

Whale: $\frac{1}{4} \times 72 = 72 \div 4 = 18$

3 **a** greater   **b** less

     c greater   **d** less

4 **a** **i** $\frac{30}{360} = \frac{1}{12}$ **ii** $\frac{60}{360} = \frac{1}{6}$

     b **i** halved

        ii stayed the same

        iii doubled

     c 2018: $\frac{1}{12} \times 144 = 12$ and 2019: $\frac{1}{6} \times 72 = 12$

     d 2018: $\frac{120}{360} = \frac{1}{3}$, $\frac{1}{3} \times 144 = 48$ and 2019: $\frac{60}{360} = \frac{1}{6}$,

       $\frac{1}{6} \times 72 = 12$ and $4 \times 12 = 48$

     e 26

5 **a** 60   **b** 60

     c 64. Women = 126, men = 62.

     d There are more women than men in the survey, so when they have the same angles in the pie charts the women's sector must represent more than the men's sector.

6 Wiston Gym has the larger number of members choosing cross-trainer as their favourite equipment.

   Wiston Gym 30% × 190 = 3 × 19 = 57

   Crundale Gym 45% × 120 = 4.5 × 12 = 54

7 **a** Fossil fuels: Argentina = 68%, Brazil = 17%, Chile = 60%

      4 × 17% = 68%, 3 × 17% = 51% and 60% > 51%

     b learners' statements

      Example: Looking at the percentages of electricity produced by hydroelectric plants, the percentage in Argentina is the same as the percentage in Chile, and the percentage in Brazil is more than two times the percentage in Argentina and Chile.

     c learners' statements

      Example: Looking at the percentages of electricity produced from other renewable sources, the percentage in Chile is five times the percentage in Argentina, and the percentage in Brazil is six times the percentage in Argentina.

     d Argentina = 1.2 kW, Brazil = 27 kW and Chile = 3.6 kW.

     e Marcus is correct.

   The number in Brazil is more than 22 times the number in Argentina because 22 × 1.2 = 26.4 kW and 27 > 26.4

      The number in Chile is exactly 3 times the number in Argentina because 3 × 1.2 = 3.6 kW

8 **a** 180   **b** 3600   **c** 2700

**9** **a** 21   **b** 72

Exercise 16.5

1 **a** compound bar chart as it will show the totals for two different days

     b pie chart as it shows proportions

     c scatter graph as it shows two sets of data points

     d stem-and-leaf diagram as it will show the numbers in order

2 **a**
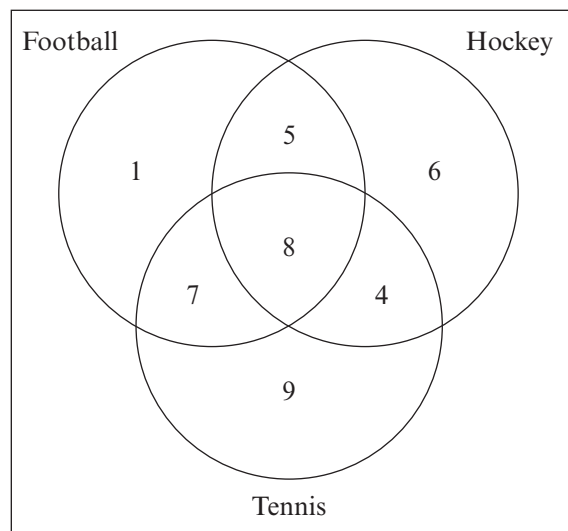
     b Clearly shows all the information.

     c learners' comments
      Example: 8 girls played all three sports.

3 **a**
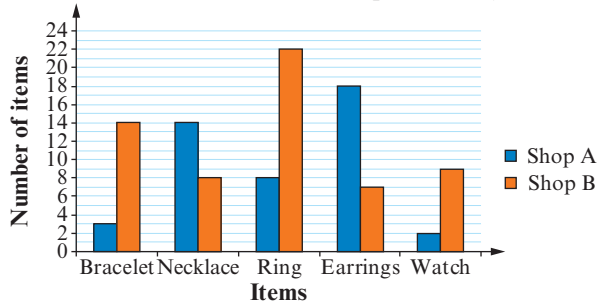
Number of times people exercised in one month
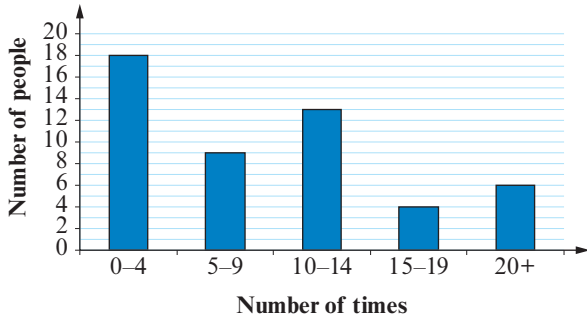
b clearly shows the frequencies for discrete data

c learners' comments

Example: 0–4 times was the most popular number of times that people exercised in one month.
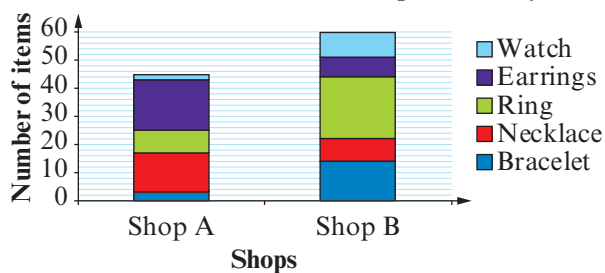
4 **a** **i**

Number of items sold in two shops on one day

ii

Number of items sold in two shops on one day

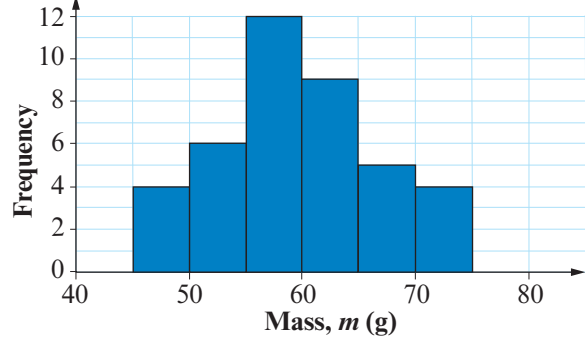**b** **i** individual number of item sales

ii total number of item sales

c **i** total number of item sales

ii individual number of item sales

d **i** dual bar chart

ii compound bar chart

5 **a**

Mass of eggs laid by Shania's chickens

b clearly shows frequencies using continuous data

c learners' comments, for example: 55 g−60 g was the most frequent mass of egg

6 **a** Key: 1 | 1 means 11 minutes

1	1 5 7 8 8 8 9 9
2	1 2 5 5 6
3	0 2 3 5 6 9
4	0 1 2 5 8

b It shows all the times, as well as showing them in order and in their groups.
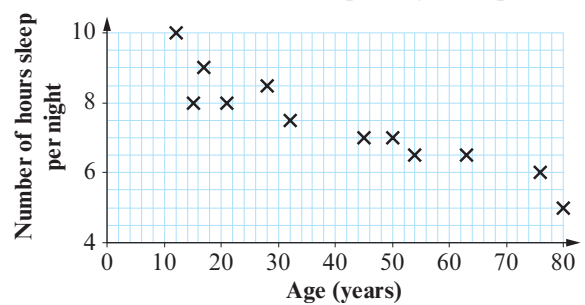
c learners' comments

d **i** 18 minutes **ii** 25.5 minutes

iii 37 minutes.

7 **a**

Number of hours of sleep and age of 12 patients

b shows the two sets of data points and any relationship between them

c learners' comments

Example: The older you get, the less sleep you need.

d **i** 41 years old (nearest year)

ii 7.4 hours (1 d.p.)

e Yes, because it sits nicely in the middle of the data, and all the values are used.

8 learners' posters showing some or all of the information given in the tables

Exercise 16.6

1 **a** Mean: Catalonia 3.625, Andalucia 2

b Catalonia scored more goals, on average, per match.

c Range: Catalonia 4, Andalucia 6

d Andalucia's scores were more varied.

2 **a** **i** Males: 21 g, 22 g, 23 g, 24 g, 27 g, 27 g, 28 g, 28 g, 29 g

Females: 18 g, 19 g, 20 g, 21 g, 22 g, 24 g, 30 g, 32 g

ii Median: Males 27 g, Females 21.5 g

iii Range: Males 8 g, Females 14 g

b The males are heavier, on average.

c The males, masses are less varied.

3 **a** **i** Boys: 0, 0, 0, 1, 1, 1, 1, 1, 2, 2, 3, 3, 4

Girls: 0, 0, 0, 0, 0, 0, 0, 0, 1, 1, 2, 2, 3, 3, 5

ii mode: Boys: 1, Girls: 0

iii range: Boys: 4, Girls: 5

b The boys missed more school days, on average.

c The number of school days missed by the girls is more varied.

4 The girls did better on average. Mean for girls is 23. Mean for boys is 21.

5 **a** Spanish because the mean is greatest

b English as it has the greatest range

c Youngest person in each class: Japanese: 24, Spanish 32, English 19

Example: Worked out age of oldest person – age range

6 **a**

	Mean	Median	Mode	Range
Pablo	70	72.5	64	11
Carlos	71	71.5	72	4

b **i, ii** learners' choices and explanations
Example: Pablo did better on average as his mean and mode are lower. His median is slightly higher than Carlos's but this doesn't take into account all of his scores.

c Carlos had more consistent scores as his range is lower.

7 **a**

	Mean	Median	Mode	Range
March	49	42	40	35
April	46	45	–	15

b **i** True if you use the mean, but false if you use the median.

ii True as the range is greater in March than April.

c You can work out the mode for March, but not for April as all the values are different.

8 **a**

	Mean	Median	Mode	Range
Men 2010	2.27	2	1	7
Women 2011	2.69	2.5	3	6

b learners' choices and explanations

Example: The mean is the best average to use because it uses all the scores.

c Example: The mean for men is 2.27 and for women is 2.69. The women's average is better but there is little difference between the two.

d The number of goals scored per match was more varied for the men than for the women. The range is greater for the men than for the women.

9 learners' descriptions

Example: The average age of the men is about 9 years more than the average age of the women (using the median and mean). The range of the men's ages is 48 years and of the women's ages is 34 years. There is more variation in the men's ages.

10

	Mean	Median	Mode
Website A	3.2	3	2
Website B	3.04	4	1

learners' choices and explanations

Example: Website A is better as it has a higher mean and mode. Also 35% of website B's users said it was poor.