> Learner's Book answers

Unit 1

Getting started

1 **a** 2, 3, 5, 7, 11, 13, 17, 19

  b The even numbers 22, 24, 26, 28 are not prime. 21 and 27 are multiples of 3. 25 is a multiple of 5. That just leaves 23 and 29 as prime numbers.

2 **a** 1, 2, 3, 6, 9, 18

  b 18, 36, 54, 72, 90

  c 6

  d 36

3 **a** -3 **b** -9 **c** -18

  d -2 **e** -2 **f** 4

4 **a** square   **b** cube

  c cube   **d** both square and cube

  e square   **f** square

5 **a** 10   **b** 5   **c** 9

Exercise 1.1

1 **a**, **b**, **c** and **d** Many different trees are possible. They all end with 2, 2, 2, 3, 5.

2 **a** and **b** Different trees are possible but they should end with 2, 2, 3, 3, 3.

  c $108 = 2^2 \times 3^3$

  d peer discussion

3 **a** Different trees are possible.

  b $200 = 2^3 \times 5^2$

  c peer discussion

  d There are two different possible trees.

4 **a** Many trees are possible.

  b $330 = 2 \times 3 \times 5 \times 11$

5 **a** 20 → **i** $2^2 \times 5$

  b 24 → **ii** $2 \times 3 \times 7$

  c 42 → **iii** $2^2 \times 3^2 \times 5$

  d 50 → **iv** 2×5^2

  e 180 → **v** $2^3 \times 3$

6 **a** 315   **b** 1000   **c** 396

  d 784   **e** 867

7 **a** $2^2 \times 7$ **b** $2^2 \times 3 \times 5$ **c** $2^3 \times 3^2$

  d $3^2 \times 17$ **e** $2 \times 5 \times 19$ **f** $5^2 \times 11$

8 **a** and **b**

Number	Product of primes
35	5 × 7
70	2 × 5 × 7
140	2^2 × 5 × 7
280	2^3 × 5 × 7
560	2^4 × 5 × 7
1120	2^5 × 5 × 7
. . .	. . .

9 **a** $7 \times 11 \times 13$ **b** $2^2 \times 7 \times 11 \times 13$

  c $2 \times 3 \times 7 \times 11 \times 13$

10 **a** $132 = 2^2 \times 3 \times 11$ **b** $150 = 2 \times 3 \times 5^2$

  c $2^3 \times 3^2 \times 5^2 \times 11$

11 **a** **i** 3×5 **ii** $3^2 \times 5^2$

    iii $2^2 \times 7$ **iv** $2^4 \times 7^2$

    v $2^2 \times 3^2$ **vi** $2^4 \times 3^4$

  b The indices for n^2 are double the indices for n.

  c $96^2 = 2^5 \times 3 \times 2^5 \times 3 = 2^{10} \times 3^2$. Double the indices for each factor. This method will work for all numbers.

12 a 4   **b** 280

13 a 30   **b** 900

14 a 18   **b** 540

15 a $3^2 \times 5$ **b** 3×5^2

    c 225   **d** 15

16 a 1260, many trees are possible.

    b peer discussion

17 a $2^2 \times 3^2 \times 11$ **b** $2^3 \times 3 \times 7$

    c 12   **d** 5544

18 a 2   **b** 986

19 $63 = 3^2 \times 7$ and $110 = 2 \times 5 \times 11$; they have no common prime factors so the HCF is 1.

20 a 1   **b** 1739

    c If x and y are different prime numbers, the HCF is 1 and the LCM is xy

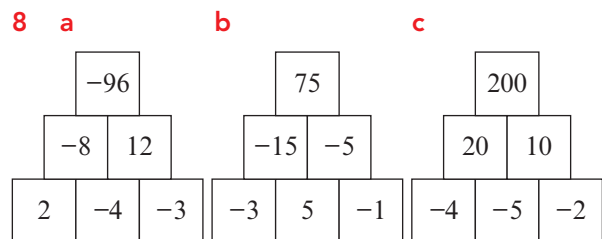   **d** peer discussion

Exercise 1.2

1 a $-3 \times 4 = -12$; $-3 \times 3 = -9$;
$-3 \times 2 = -6$; $-3 \times 1 = -3$;
$-3 \times 0 = 0$; $-3 \times -1 = 3$; $-3 \times -2 = 6$;
$-3 \times -3 = 9$; $-3 \times -4 = 12$ and so on.

    b The first number is always −3. The second number goes down by 1 each time. The answer goes up by 3 each time.

    c $-5 \times 4 = -20$; $-5 \times 3 = -15$;
$-5 \times 2 = -10$; $-5 \times 1 = -5$;
$-5 \times 0 = 0$; $-5 \times -1 = 5$; $-5 \times -2 = 10$;
$-5 \times -3 = 15$; $-5 \times -4 = 20$ and so on. The first number is always −5. The second number goes down by 1 each time. The answer goes up by 5 each time.

    d The product of two negative numbers is the same as the product of the corresponding positive numbers. For example, $-6 \times -4 = 6 \times 4 = 24$. You could write this as $-a \times -b = a \times b$.

    e learners' own sequence

    f peer discussion

2 a −10   **b** −10

    c 10   **d** 10

3 a 24   **b** 49

    c 60   **d** 88

4

×	−5	3	−8
4	−20	12	−32
−3	15	−9	24
−6	30	−18	48

5 a −32   **b** 48

    c 12   **d** −30

6 a −28   **b** −33

    c 36   **d** 25

7 a $3 \times -4 = 2 \times -6 = -12$ and
$-6 \times -2 = 12 \times 1 = -4 \times -3 = -12 \times -1 = 12$

    b There are many possible answers.

8 a   **b**   **c**

	−96	
−8		12
2	−4	−3

	75	
−15		−5
−3	5	−1

	200	
20		10
−4	−5	−2

9 a

	90	
−6		−15
−2	3	−5

    b Zara is incorrect. One possible statement is that the top number is 90, −150 or −60.

10 a 4   **b** −9

    c −4   **d** −8

11 a $-24 \div 6 = -4$ or $-24 \div -4 = 6$

    b learners' examples

    c $14 \div -2 = -7$ or $14 \div -7 = -2$

    d learners' examples

    e learners' own conjectures

    f peer discussion

12 a −3   **b** 7

    c −5   **d** 3

    e −7   **f** 6

    g −12   **h** 5

13 a   **b**   **c**

−30

−5	6

5	−1	−6

−96

12	−8

−6	−2	4

−200

10	−20

2	5	−4

14 a 6   **b** 6

     c −3   **d** −3

15 a $x = -32$ **b** $x = 45$

     c $x = -8$ **d** $x = -5$

16 a −3   **b** −3

     c 7   **d** 5

17 a 1500   **b** −1200

     c 7   **d** −5

Exercise 1.3

1 a 49   **b** 49

     c 343   **d** −343

2 a 5   **b** −3

     c −1   **d** −2

3 a $x = 10$ or -10 **b** $x = 12$ or -12

     c $x = 1$ or -1 **d** $x = 0$

     e no solution

4 a $x = 6$ **b** $x = -3$

     c $x = -1$ **d** $x = -5$

5 a 27   **b** no solution

     c 9   **d** −9

6 a $\sqrt{8^2} = \sqrt{64} = 8$ as the calculator only gives the positive square root, and $\sqrt{(-8)^2} = \sqrt{-8 \times -8} = \sqrt{64} = 8$, so the difference is 0.

     b $\sqrt[3]{4^3} - \sqrt[3]{(-4)^3} = 4 - (-4) = 4 + 4 = 8$

7 The integer is 10 or −10 so the cube is $10^3 = 1000$ or $(-10)^3 = -1000$

8 a $3 \times 13 = 39$ **b** $x = 39$ or -39

9 a $-5^2 = -25$ but $(-5)^2 = +25$

     b There is no difference: $-5^3 = -(5 \times 5 \times 5) = -125$ and $(-5)^3 = -5 \times -5 \times -5 = -125$

10 a $3^2 + 4^2 = 9 + 16 = 25 = 5^2$

     b i True: $(-3)^2 + (-4)^2 = 9 + 16 = 25 = (-5)^2$

       ii True: $12^2 + (-5)^2 = 144 + 25 = 169 = (-13)^2$

       iii False: $8^2 = 64$ but $-10^2 - 6^2 = -100 - 36 = -136$ which is not the same.

     c peer discussion

11 a i $2^2 + 2 = 4 + 2 = 6$

       ii $(-3)^2 + (-3) = 9 - 3 = 6$

     b i $3^2 + 3 = 9 + 3 = 12$

       ii $x = -4$

     c $x = 4$ or -5

     d One solution is a positive integer n and another is $-(n + 1)$.

     Another equation like this is $x^2 + x = 30$ which has the solution $x = 5$ or -6

     e peer discussion

12 a

x	$x - 1$	$x^3 - 1$	$x^2 + x + 1$
2	1	7	7
3	2	26	13
4	3	63	21
5	4	124	31

     b The third column divided by the fourth column = the second column.

     That is $(x^2 + x + 1) \div (x^3 - 1) = x - 1$

     c The next row is 6, 5, 215, 43 and $215 \div 43 = 5$

     d The result is the same for negative values of x.

     e peer discussion

13 a 5, −500, 16   **b** 5, 16

     c all six numbers: $5, -\frac{1}{5}, -500, 16, -4.8, 99\frac{1}{2}$

14 a and b

Learners may omit $2\frac{1}{2}$ from the Venn diagram.

c and d

e Peer discussion may lead to argument about 6.3 which is rational because it can be written as $6\frac{3}{10}$.

Exercise 1.4

1

Power	2^0	2^1	2^2	2^3	2^4	2^5	2^6	2^7	2^8	2^9	2^{10}
Number	1	2	4	8	16	32	64	128	256	512	1024

2

Power	3^0	3^1	3^2	3^3	3^4	3^5	3^6	3^7	3^8
Number	1	3	9	27	81	243	729	2187	6561

3 a i $2^3 \times 2^2 = 2^5$ ii $2^4 \times 2^3 = 2^7$

iii $2^2 \times 2^5 = 2^7$ iv $2^1 \times 2^7 = 2^8$

v $2^4 \times 2^5 = 2^9$

b Add the indices to get the index of the answer. Learners' own examples.

c The same rule applies. Learners' own examples.

d The rule works for powers of any positive integer.

4 a 6^5 **b** 6^5 **c** 6^7 **d** 6^6

5 a 10^5 **b** 20^6 **c** 15^6 **d** 5^8

6 a $6561 \times 3 = 19\,683$ **b** $15\,625 \times 5 = 78\,125$

7 a 3^2 **b** 9^5 **c** 12^2 **d** 15^7

8 The first part is correct because $4^2 = 16 = 2 \times 2 \times 2 \times 2 = 2^4$.
The second part is not correct because $4^3 = 4 \times 4 \times 4 = 64$ but $3^4 = 3 \times 3 \times 3 \times 3 = 81$

9 a 10^9 **b** 10^{12}

10 a 2^6 **b** 3^9 **c** 5^7 **d** 10^9

11 a 3^6

b i 2^6 ii 5^6 iii 4^6

iv 15^8 v 10^{12}

c i N^6 ii N^8 iii N^{15}

d To find a power of a power you multiply the indices $\left(N^a\right)^b = N^{ab}$

12 a i $2^6 \div 2^2 = 2^4$ ii $3^4 \div 3^1 = 3^3$

iii $2^9 \div 2^4 = 2^5$ iv $3^6 \div 3^2 = 3^4$

v $3^2 \div 3^2 = 3^0$

b learners' own examples

c When you divide powers you subtract the indices.

d peer discussion

13 a 2^2 **b** 10^3 **c** 15^2

d 8^1 or 8 **e** 2^4 **f** 2^0 or 1

14 a 9^7 **b** 9^3 **c** 9^{10}

d 5^9 **e** 12^5 **f** 7^9

g 10^0 or 1

15 a Yes: $\left(5^2\right)^3 = 5^2 \times 5^2 \times 5^2 = 5 \times 5 \times 5 \times 5 \times 5 \times 5$

$= (5 \times 5 \times 5) \times (5 \times 5 \times 5) = \left(5^3\right)^2$

b Yes

16 a $3^2 \times 5^2$ **b** $3^3 \times 5^3$

c $3^5 \times 5^5$ **d** $3^8 \times 5^8$

17 a 5^2 **b** 5^0

c You might suggest 5^{-2}.

Check your progress

1 a Different trees are possible. They all end with 2, 5, 5, 7

b $350 = 2 \times 5^2 \times 7$

c $112 = 2^4 \times 7$

d 14

e 2800

2

×	−6	−5	7
−10	60	50	−70
3	−18	−15	21
−7	42	35	−49

3 **a** No: $(-5)^2 = 25$

  b No: $-9 \times -11 = 99$

  c No: $45 \div -9 = -5$

  d Yes

4 **a** −8   **b** 6

  c −20   **d** −10

5 **a** $x = 6$ or −6   **b** No solution

  c $x = 2$ **d** $x = -3$

6 **a** 3   **b** 0

7 **a** 36   **b** −18

8 **a** 8^5 **b** 8^4 **c** 8^0 **d** 8^9

9 **a** 2^{12} **b** 3^8

Unit 2

Getting started

1 **a** $2n$ **b** $n + 5$

2 12

3 **a** $7c + 7d$ **b** $2xy + 8yz$

4 **a** $4x + 12$ **b** $12 - 18y$

5 **a** $n = 3$ **b** $m = 9$

  c $p = 9$ **d** $r = 6$

6 $x \geq 3$

Exercise 2.1

1 In the <u>expression</u> $4x + 9$, x is a <u>variable</u>, $4x$ and 9 are <u>terms</u> of the expression. 4 is the <u>coefficient</u> of x. 9 is a <u>constant</u>. The expression is not equal to anything so cannot be <u>solved</u>.

2 **a** **i** $x - 2$ **ii** $x + 2$

    iii $\frac{x}{2}$ **iv** $2x$

  b **i** 10   **ii** 14

    iii 6   **iv** 24

3 **a** **i** $6n + 1$ **ii** $\frac{n}{4} + 5$

    iii $2n - 3$ **iv** $\frac{n}{10} - 7$

  b **i** 121   **ii** 10

    iii 37   **iv** −5

4 a and iv,   b and i,

  c and v,   d and ii,

  e and vii,   f and iii

  vi: subtract n from 4, then multiply by 5

5 learners' answers including Sofia is correct

6 **a** $\frac{x}{3} + 1$ **b** $\frac{x+1}{3}$

  c $\frac{x-1}{3}$ **d** $\frac{x}{3} - 1$

7 Learners' answers including they are all correct. The usual convention is to write the expression without a multiplication sign (i.e. like Zara and Arun, not like Sofia).

8 **a** Equivalent to $\frac{3x}{4}$ are: A, E, F and H

    Equivalent to $\frac{4x}{3}$ are: C, D and I

    Equivalent to $\frac{x+3}{4}$ are: B and J

  b G: $\frac{3}{4} + x$

9 **a** correct

  b incorrect, should be $5 - \frac{2y}{5}$ or $5 - \frac{2}{5}y$

10 **a** **i** $\frac{x}{2} + 8$ or $\frac{1}{2}x + 8$

    ii $\frac{3x}{4} - 12$ or $\frac{3}{4}x - 12$

    iii $7 + \frac{4x}{5}$ or $7 + \frac{4}{5}x$

    iv $20 - \frac{5x}{9}$ or $20 - \frac{5}{9}x$

  b **i** one-sixth of x add 2

    ii five-sevenths of x subtract 4

    iii eight subtract two-thirds of x

    iv three add seven-eighths of x

11 **a** perimeter is $a + 12b$ cm   area is $3ab$ cm²

  b perimeter is $14c + \frac{6}{5}d$ cm   area is $\frac{21}{5}cd$ cm²

12 $4y + 9$

13 **a** $c + 3s$ **b** $3c + 4g + 6s$

14 a $a+\dfrac{b}{2}$ **b** $2b+\dfrac{3c}{4}$

c $3a+\dfrac{b}{4}+\dfrac{4c}{5}$

15 a $6\left(\dfrac{y}{2}+5\right)$ **b** $2\left(\dfrac{y}{5}+6\right)$

c $5\left(\dfrac{5y}{6}+2\right)$ **d** $6\left(\dfrac{2y}{5}+5\right)$

Exercise 2.2

1 a 2   **b** -2

c -18 **d** -5

e 3   **f** -7

2 a -21 **b** 4

c 23   **d** -7

e -3 **f** 2

3 a 21   **b** -15

c 45   **d** -15

e 16   **f** 3

g 54   **h** 3

i -44 **j** 8

4 a $-3 \times -3 = 9$ not -9

b 1   **c** 29

5 a She must work out $(-2)^3$ first before multiplying by 5.

b -40 **c** -54

6 a i months = years × 12

ii $m = 12y$

b $m = 96$

7 a i cost = 6 + kilometres × 2

ii $c = 6 + 2k$

b $c = 76$

8 a $v = 125$

b $v = 158$

c $v = 200$

9 a $F = 12$

b $F = 54$

c $F = -32$

10 a 145 cm   **b** 157.5 cm

c 132.5 cm   **d** 175 cm

e 160 cm   **f** 120 cm

11 a 60 m or 57 m   **b** 59.7 m

12 Prism A $V = 360\,\text{cm}^3$, Prism B $V = 378\,\text{cm}^3$. No, Xavier is wrong. Prism B has the larger volume by 18 cm³.

13 a B   **b** C

c A   **d** B

e A

14 a $T = 45$ **b** $m = \dfrac{T}{g}$

c $m = 32$

15 a $h = 35$ **b** $k = h + d$

c $k = 2.25$

16 a $f = 5$ **b** $w = fp$

c $w = 13$

17 a learners' answers

b learners' answers

c i $p = 9$ **ii** $m = 6$

Exercise 2.3

1 a $3x + 12$ **b** $8y - 16$

c $27q - 36$

2 a $4x + 24$ **b** $7z - 14$

c $2a + 16$ **d** $18 - 24e$

e $4p + 6q$ **f** $54t - 18s$

g $42xy - 14z$ **h** $10x + 5y + 20$

3 a $xy + 3x$ **b** $y^2 - 2y$

c $3p + 4p^2$ **d** $6q^2 - 15q$

4 a $y^2 + 8y$ **b** $2wz - z$

c $m^2 - 4m$ **d** $2n^2 + 5n$

e $9n - 8n^2$ **f** $a - 3ab$

g $2e^2 + 7ef$ **h** $3gh + 7g^2$

i $2h^2 - 5hk$ **j** $3cd - 5de$

5 learners' answers

Both Zara and Arun are correct but convention is that we write the letters in alphabetical order like Zara.

6 a learners' answers

b learners' answers

c **i** $2x^2 + 6xy$ **ii** $15y^2 + 18y$

iii $24b^2 - 8ab$ **iv** $4f^2 + 2fg - 6f$

7 $16x^3 + 12x^2$: A, E, I

$30x^3 + 20x$: B, D, G

$24x^3 + 18x^2$: C, F, H

8 a $x(2x+5) - 3x(2x+4) = 2x^2 + 5x - 6x^2 - 12x$

b $x(2x+5) - 3x(2x-4) = 2x^2 + 5x - 6x^2 + 12x$

c
☺(✿ + ❀) + ◯(★ + ◆) = ☺✿ + ☺❀ + ◯★ + ◯◆
☺(✿ + ❀) + ◯(★ − ◆) = ☺✿ + ☺❀ + ◯★ − ◯◆
☺(✿ + ❀) − ◯(★ + ◆) = ☺✿ + ☺❀ − ◯★ − ◯◆
☺(✿ + ❀) − ◯(★ − ◆) = ☺✿ + ☺❀ − ◯★ + ◯◆

9 a $2x^2 + 7x$ **b** $6z^2 + 6z$

c $u^2 + 2u$ **d** $2w^2 + 20wx$

10 a Q1: the $+21$ should be -21

Q2: up to $ac + 3bc$ is correct, but this cannot be simplified as they are not like terms

Q3: the $9x^2$ should be $3x^2$

b Q1: $2x + 19$

Q2: $ac + 3bc$

Q3: $3x^2 + 2y^2 + 14xy$

Activity 2.3

Correct expansions are:

A $17x^2 + 5x$

B $7y^3 + 48y^2 + 4y$

C $14p^3 + 49p^2 + 2p$

D $15k^3 - 6k + 18$

E $3n^3 - 4n^2 - 20n$

F $30m$

Exercise 2.4

1 a $3(x+5)$ **b** $5(2y-3)$

c $7(2-4x)$ **d** $3(4-3y)$

2 a $x(4x+5)$ **b** $6y(x+2)$

c $7y(1-y)$ **d** $3x(7-4y)$

3 peer discussion, e.g.

Arun has fully factorised $6x+18$ to get $6(x+3)$.

Marcus has only partially factorised $6x+18$ to get $3(2x+6)$.

Marcus has used a common factor, but Arun has used the highest common factor.

4 a $2(x+2)$ **b** $2(2b-3)$

c $2(4+5y)$ **d** $2(9-10m)$

5 a $3(6+7p)$ **b** $3(y-6)$

c $3(3+5m)$ **d** $3(4-9x)$

6 a $5(2z+1)$ **b** $4(2a-1)$

c $7(2+3x)$ **d** $6(3-4z)$

7 a peer discussion; Sofia is correct.

b **i** $4y$

ii $3p$

iii a

8 A and iii, B and i, C and iv, D and ii

9 a $x(3x+1)$ **b** $6y(y-2)$

c $3b(1+3b)$ **d** $3n(4-5n)$

e $9(2y-x)$ **f** $3(4y+3x)$

g $4y(2x-1)$ **h** $5z(3+2y)$

10 a $2(x+3y+4)$ **b** $4(y-2+x)$

c $3(3xy+4y-5)$ **d** $x(5x+2+y)$

e $y(9-y-x)$ **f** $3y(y-3+2x)$

11 $5(2x+6) + 2(3x-5) = 10x + 30 + 6x - 10$
$$= 16x + 20$$
$$= 4(4x + 5)$$

12 Correct expansion is:

$$6(3y+2)-4(y-2)=18y+12-4y+8$$
$$=14y+20$$
$$=2(7y+10)$$

Marcus has used -8 instead of $+8$ in the first line of the expansion like this:

$$6(3y+2)-4(y-2)=18y+12-4y-8$$
$$=14y+4$$
$$=2(7y+2)$$

13 A length $=a+9$ **B** length $=4d-5c$

Exercise 2.5

1 a equation   **b** expression

   c formula   **d** expression

2 a $x=7$ **b** $x=9$

   c $y=44$ **d** $y=8$

3 a $\quad\frac{x}{2}-3=15$

   $\quad\frac{x}{2}-3+3=15+3$

   $\quad\frac{x}{2}=18$

   $\quad x=18\times2$

   $\quad x=36$

   b $\quad\frac{x}{3}+1=12$

   $\quad\frac{x}{3}+1-1=12-1$

   $\quad\frac{x}{3}=11$

   $\quad x=11\times3$

   $\quad x=33$

   c $\quad\frac{x}{4}+9=30$

   $\quad\frac{x}{4}+9-9=30-9$

   $\quad\frac{x}{4}=21$

   $\quad x=21\times4$

   $\quad x=84$

4 learners' answers

Example: It doesn't matter as you will get the same answer, but it is easier to have the greater number of 'y's on the left hand side of the equation.

5 a $x=2$ cm   **b** $x=14$ cm

   c $x=12$ cm

6 learners' answers

Example: Substitute the value for x into the expression for each side length of the triangles and the answers should be equal.

7 a $y=7$ cm and side lengths $=16$ cm

   b $y=4$ cm and side lengths $=27$ cm

   c $y=12$ cm and side lengths $=36$ cm

8 a $x=8$, $y=7$ **b** $x=9$, $y=5$

   c $x=7$, $y=4$ **d** $x=32$, $y=3$

To check answers, substitute values of x and y into expressions for side lengths.

9 a $3x+8=23$, $x=5$

   b $\frac{x}{4}-8=5$, $x=52$

   c $5x-4=2x+20$, $x=8$

   d $2(x+5)=5x-14$, $x=8$

10 a $6n+2n+5+n-5=180$ or $9n=180$

   b $n=20$

   c $6n=120°$, $2n+5=45°$, $n-5=15°$

11 a $4x-6=2x+18$

   b $x=12$

   c $4x-6$ and $2x+18$ both equal 42°, 3rd angle $=96°$

12 a $x=2$ **b** $x=3$

   c $y=12$ **d** $y=30$

13 a $a=-4$ **b** $c=3.5$

   c $d=5$

14 a $\frac{y}{4}-18=4$, $y=88$

   b $2y+14=-20$, $y=-17$

Table shows all possible values for y.

	4	−2	−20
$2y+14$	$y=-5$	$y=-8$	$y=-17$
$8(y-12)$	$y=12.5$	$y=11.75$	$y=9.5$
$\frac{y}{4}-18$	$y=88$	$y=64$	$y=-8$

Exercise 2.6

1 **a** x is greater than 6 and less than 11

b x is greater than or equal to 12 and less than or equal to 18

c x is greater than 0 and less than or equal to 20

d x is greater than or equal to -9 and less than -1

2 **a** $3 \leqslant y < 17$ **b** $15 < y < 25$

c $-2 < y \leqslant 5$ **d** $-9 \leqslant y \leqslant -3$

3 **a**
```
  ○──────○
  3 4 5 6 7 8
```
b
```
  ●──────●
  5 6 7 8 9 10
```
c
```
  ○─────────●
  −5 −4 −3 −2 −1 0 1 2
```
d
```
  ●─────────○
  −3 −2 −1 0 1 2 3
```
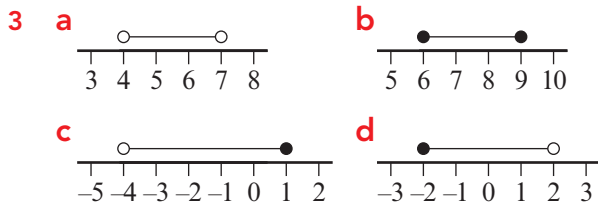
4 **a** $12 < x < 16$ **b** $1 < x \leqslant 5$

c $-3 \leqslant x < 1$ **d** $2 \leqslant x \leqslant 8$

5 **a** Sofia: $x > 5, 2 \times x > 2 \times 5, 2x > 10$
Zara: $x > 5, x - 2 > 5 - 2, x - 2 > 3$

b learners' answers
Examples: $3x > 15, 4x > 20, 10x > 50,$
$x - 3 > 2, x - 5 > 0, x + 5 > 10$

c learners' answers
Example: It is not possible to say as there is an infinite number of possibilities.

6 **a** $x > 8$ is equivalent to $3x > 24$

b $x < 3$ is equivalent to $5x < 15$

c $y \geqslant 7$ is equivalent to $y + 3 \geqslant 10$

d $y \leqslant 2$ is equivalent to $y - 4 \leqslant -2$

7 **a** Ryan has misunderstood the symbols: he has interpreted $\leqslant$ as 'greater than' and $<$ as 'less than or equal to'.

i smallest integer is 12

ii largest integer is 17

iii x could be 12, 13, 14, 15, 16, 17

b peer discussion

8 **a** **i** 4   **ii** 7

iii 4, 5, 6, 7

b **i** 5   **ii** 7

iii 5, 6, 7

c **i** 0   **ii** 5

iii 0, 1, 2, 3, 4, 5

d **i** -10 **ii** -6

iii $-10, -9, -8, -7, -6$

9 peer discussion, including Arun is correct.
Convention is that we would write $2 < y < 9$ rather than $9 > y > 2$.

10 **a** T   **b** F   **c** T   **d** F

11 **a** **i** $8 \leqslant m < 15$ **ii** $7 < m \leqslant 10$

iii $0 < m < 6$

b No – if you show these inequalities on a number line they are two separate sections that cannot be combined.

12 **a** smallest integer is 3 not 2; m could be 3, 4, 5, 6, 7 (but not 2)

b **A** **i** 6   **ii** 9   **iii** 6, 7, 8, 9

B **i** -6 **ii** -3 **iii** $-6, -5, -4, -3$

13 **a** learners' answers

b answers arranged into rows

Inequality	Smallest integer	Largest integer	List of integers
$1.5 \leqslant x \leqslant 4$	2	4	2, 3, 4
$0.8 < x < 5.9$	1	5	1, 2, 3, 4, 5
$3 < x \leqslant 6.1$	4	6	4, 5, 6
$2.2 \leqslant x < 3.9$	3	3	3
$-4.5 < x < 1.1$	-4	1	$-4, -3, -2, -1, 0, 1$
$-5.01 < x \leqslant 0$	-5	0	$-5, -4, -3, -2, -1, 0$

Check your progress

1 $\dfrac{x}{2} + 5$

2 **a** $K = 48$ **b** $m = \dfrac{K}{g}$

c $m = 7.5$

3 **a** $x^2 + 3x$ **b** $35y^2 - 20wy$

4 **a** $3(2x + 3)$ **b** $2y(y - 6)$

5 $x = 5, y = 12$

6 $5 < x \leqslant 20$

Unit 3

Getting started

1 **a** 45   **b** 180

    c 8200   **d** 460

2 **a** 7   **b** 34.2

    c 1.4   **d** 31.2

3 **a** A 7.2   **b** B 12.5   **c** B 0.8

4 **a** 4.59   **b** 0.6723

    c 54.789   **d** 12.050 30

Exercise 3.1

1 **a** 2   **b** 20   **c** 200   **d** 0.2

2 **a** 4   **b** 400   **c** 0.4   **d** 40

3 **a** learners' answers

    b **Sofia:** When I multiply 56 by 0.01, I move the digits 5 and 6 two places to the right in the place value table. This gives me an answer of 0.56

       Arun: When I multiply 56 by 0.01, I move the decimal point two places to the left. This gives me an answer of 0.56

4 **a** 6.2   **b** 5.5   **c** 12.5

    d 0.32   **e** 0.37   **f** 6.55

    g 7.5   **h** 0.04

5 **a** 20   **b** 200

    c 2000   **d** 2

6 **a** 400   **b** 4000

    c 40 000   **d** 40

7 **a** learners' answers

    b $0.45 \div 0.1 = 4.5$ and $78 \div 0.01 = 7800$

8 **a** 70   **b** 45   **c** 5220

    d 6.7   **e** 200   **f** 850

    g 32   **h** 722.5

9 **a** 1.8   **b** 0.236

    c 6   **d** 450

10 **a** ÷   **b** ×   **c** ×

    d ×   **e** ÷   **f** ÷

11 **a** 0.01   **b** 0.1   **c** 0.01

    d 0.1   **e** 0.1   **f** 0.1

12 **a** 12.5 g   **b** 0.8 g

    c Yes, multiplying by 0.1 is the same as finding 10%.

    d 1%. Multiplying by 0.01 is the same as finding 1%.

13 **a** A, F and J all equal 2.4, B, E and H all equal 24, C, G and I all equal 240

    b D is left over and equals 2400. Any two calculations that give 2400, e.g. $240 \div 0.1$, $24\,000 \times 0.1$ or $240\,000 \times 0.01$

14 125

15 **a** learners' answers

       Example: $-4 \times 0.1 = -0.4$, which is not greater than zero

    b learners' answers

       Example: $0.4 \div 0.01 = 40$, which is not greater than 100

Exercise 3.2

1 **a** C   **b** A

    c B   **d** C

2 **a** 240   **b** 0.24   **c** 24

    d 0.0024   **e** 2400   **f** 2.4

3 **a** learners' answers

       Example: In **a** he has forgotten the zeros. It should be 45 000. In **b** he has rounded to 2 d.p. not 2 s.f. It should be 0.033.

    b learners' answers

       Example: Fill in the gaps between the significant figures and the decimal point with zeros.

    c learners' answers

       Example: Fill in the gaps between the decimal point and the significant figures with zeros.

4 **a** 100   **b** 46 000   **c** 18.7

    d 0.09   **e** 0.79   **f** 1.409

    g 1000   **h** 0.70   **i** 8.60

5 **a** D   **b** C

    c B   **d** D

    e C

6 **a** 200   **b** 210   **c** 209

    d 209.1   **e** 209.10   **f** 209.095

7 **a** 683.615 77 31

    b **i** 700   **ii** 680

       iii 684   **iv** 683.6

       v 683.62   **vi** 683.616

8 96 000

9 0.4 g

10 298 000 000 metres per second

11 learners' answers, but convention is that answers are usually given to the same accuracy as the numbers in the question. So Sofia is correct.

12 12 600 × \$26.80 = \$337 680 which is \$338 000 to 3 s.f.

13 $a = 2.1$ to 2 s.f.

14 **a** **i** 16   **ii** 16.1 (3 s.f.)

     b **i** 700   **ii** 713 (3 s.f.)

     c **i** 40   **ii** 42.6 (3 s.f.)

     d **i** 80   **ii** 67.2 (3 s.f.)

Check your progress

1 **a** 9   **b** 55.2   **c** 1.35

    d 0.08   **e** 60   **f** 235

    g 520   **h** 68

2 B is a different answer from the others. A = 0.52, B = 520, C = 0.52, D = 0.52

3 **a** 78   **b** 0.0679

    c 1.550   **d** 12 453 000

4 0.0041

Unit 4

Getting started

1 **a** 4.5 > 4.1   **b** 6.57 < 6.68

    c 10.52 < 10.59   **d** 2.784 > 2.781

2 0.756, 0.759, 0.761, 0.763

3 **a** T   **b** F

    c F   **d** T

4 12 × 1.8 = 21.6, 19 × 1.2 = 22.8, 9 × 2.5 = 22.5, 25 × 0.87 = 21.75, 320 × 0.07 = 22.4

5 **a** 4.1   **b** 6.3

    c 25.48   **d** 2.405

Exercise 4.1

1 **a** 2.06, 5.49, 5.91, 7.99

    b 2.55, 2.87, 3.09, 3.11

    c 11.82, 11.88, 12.01, 12.1

    d 8.9, 9.09, 9.4, 9.53

2 **a** 4.23 < 4.54   **b** 6.71 > 6.03

    c 0.27 > 0.03   **d** 27.9 > 27.85

    e 8.55 > 8.508   **f** 5.055 < 5.505

3 learners' answers

4 **a** 23.592, 23.6, 23.605, 23.66

    b 0.009, 0.08, 0.1, 0.107

    c 6.007, 6.71, 6.725, 6.78

    d 11.002, 11.02, 11.032, 11.1

5 **a** 6.71 ≠ 670 ml

    b 4.05 t ≠ 4500 kg

    c 0.85 km = 850 m

    d 0.985 m ≠ 985 cm

    e 14.5 cm = 145 mm

    f 2300 g ≠ 0.23 kg

6 **a** 4.51 > 2700 ml

    b 0.45 t < 547 kg

    c 3.5 cm < 345 mm

    d 0.06 kg < 550 g

    e 7800 m > 0.8 km

    f 0.065 m < 6.7 cm

7 **a** 780 g, 1950 g, 2.18 kg, 2.3 kg

    b 0.8 cm, 9 mm, 12 mm, 5.4 cm

    c 0.5 m, 53 cm, 650 cm, 12 m

   d   95 ml, 450 ml, 0.55 l, 0.9 l

   e   780 m, 1450 m, 6.4 km, 6.55 km

   f   50 kg, 0.08 t, 0.15 t, 920 kg

8 **a** No, his list starts with the largest and ends with the smallest.

     It should be −4.52, −4.38, −4.31, −4.05

  b learners' answers

9 **a** −4.27 > −4.38   **b** −6.75 < −6.25

  c −0.2 < −0.03   **d** −8.05 > −8.9

10 **a** −4.76, −4.67, −4.5, −4.05

   b −11.91, −11.6, −11.525, −11.08,

11 **a** 25 km. It is much further than the other distances.

   b Mia is correct.

      1.64 km = longest, 0.2 km = shortest, 8 × 0.2 km = 1.6 km and 1.64 km > 1.6 km

   c Shen swims in the 25 m pool as all his distances are multiples of 25 m.

      Mia swims in the 20 m pool as all her distances are multiples of 20 m.

12 **a** A 2.5, B 2.4, C 2.3, D 2.1, E 2.25, F 2.45

   b 2.1, 2.25, 2.3, 2.4, 2.45, 2.5

13 No, there are 7 numbers not 8. x could be: 3.27, 3.28, 3.29, 3.30, 3.31, 3.32, 3.33

14 y could be: −0.273, −0.272, −0.271, −0.270

Exercise 4.2

1 **a** −0.8   **b** 0.6   **c** −2.1

   d 5.6   **e** −3.6

2 **a** −0.18   **b** −1.8

   c −0.018   **d** −18

3 C −7.65, E −7.28, A −7.2, D −7.04, B −7.02

4 learners' answers

5 **a** **i** 2 × 4 = 8

       0.2 × 4 = 0.8

       0.2 × 0.4 = 0.08

       0.2 × 0.04 = 0.008

       0.2 × 0.004 = 0.0008

     ii 3 × 5 = 15

       0.3 × 5 = 1.5

       0.3 × 0.5 = 0.15

       0.3 × 0.05 = 0.015

       0.3 × 0.005 = 0.0015

  b **i** 0.009   **ii** 0.48

     iii 0.028   **iv** 0.0015

     v 0.036   **vi** 0.0066

6

Central: 0.36

Connected bubbles: 3 × 0.12, 0.6 × 0.6, 0.9 × 0.4, 6 × 0.06, 36 × 0.01, 0.04 × 9, 4 × 0.09, 0.3 × 1.2
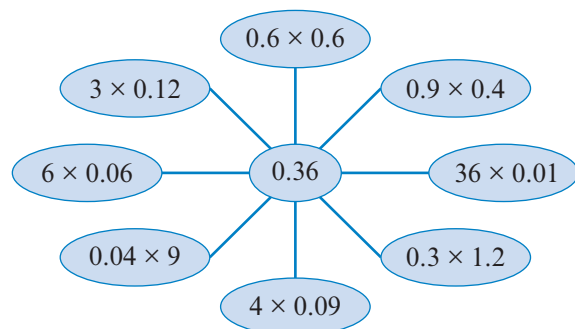
7 **a** **i** 365.4   **ii** 36.54

     iii 365.4   **iv** 36.54

  b, c learners' answers

8 **a** 158 × 46 = 7268

   b **i** 726.8   **ii** 726.8

     iii 72.68   **iv** 7.268

     v 7.268   **vi** 0.072 68

9 learners' answers

10 **a** 62.98 Estimate: 7 × 9 = 63

   b 4.648 Estimate: 0.6 × 8 = 4.8

   c 1.8745 Estimate: 0.2 × 8 = 1.6

   d 0.175 26 Estimate: 0.7 × 0.3 = 0.21

11 **a** Estimate: 0.5 × 3 = 1.5. Her answer of 12.6 must be wrong.

   b Estimate: 8 × 0.009 = 0.072. Her answer of 0.072 54 could be correct.

   c Estimate: 0.07 × 0.04 = 0.0028. Her answer of 0.027 95 must be wrong.

12 **a** 6 × 7 = 42 mg   **b** 42.34 mg

13 **a** 1 × 4 = 4 g   **b** 3.255 g

Exercise 4.3

1 a $\dfrac{24}{4} = 6$ **b** $\dfrac{72}{9} = 8$

  c $-\dfrac{420}{6} = -70$ **d** $-\dfrac{450}{5} = -90$

2 D because the answer is 8. All the others have an answer of 7.

3 learners' answers

4 a 2.3   **b** 8.2

  c −860   **d** −960

5 $1.35 per metre

6 learners' answers

7 a Estimate: $30 \div 0.3 = 100$
    Accurate: $27.6 \div 0.3 = 92$

  b Estimate: $-200 \div 0.4 = -500$
    Accurate: $-232 \div 0.4 = -580$

  c Estimate: $300 \div 1 = 300$
    Accurate: $306 \div 0.9 = 340$

  d Estimate: $-490 \div 0.7 = -700$
    Accurate: $-483 \div 0.7 = 690$

  e Estimate: $40 \div 0.8 = 50$
    Accurate: $43.76 \div 0.8 = 54.7$

  f Estimate: $-30\,000 \div 0.6 = -50\,000$
    Accurate: $-33\,972 \div 0.6 = -56\,620$

8 a She hasn't written down the 0 above the 6.

  b 42.05

9 a Carried on the division by writing a decimal point after the 7, then carrying the remainder of 9 onto the zero in the tenths column.

  b 256.5

10 a

1	2	3	4	5	6	7	8	9
19	38	57	76	95	114	133	152	171

  b 31.25   **c** $30 \times 2 = 60$

11 a

1	2	3	4	5	6	7	8	9
25	50	75	100	125	150	175	200	225

  b $23.56

  c $23.56 \approx 20$ and $20 \times 2.5 = 50$

12 14.75 m

13 a i 425   **ii** 27
     iii 4250   **iv** 270

  b learners' answers

  c i 425   **ii** 42.5
     iii 4.25   **iv** 0.425

  d learners' answers

  e peer discussion

14 a 6.3   **b** 74.86   **c** −2473.5

Exercise 4.4

1 a 0.14   **b** 0.12

2 a 0.48   **b** 0.6
  c 0.24   **d** 0.33

3 a 46.8   **b** 7.47

4 a 25.2   **b** 15.3   **c** 4.41

5 a, b learners' answers

  c i $26 \times 9.9 = 257.4$

    ii $26 \times 0.99 = 25.74$

6 $59.4\,\text{m}^2$

7 3168

8 a 327   **b** 4.6   **c** 546
  d 26.4   **e** 42

9 a 8.4   **b** 14.4

10 a 12   **b** 29

11 a 25.2   **b** 19.2
  c 45   **d** 108

12 learners' answers

13 a Abdul gets $4410, Zhi get $3150, Paula gets $2772, Yola gets $2268

  b $4410 + 3150 + 2772 + 2268 = 12\,600$

14 Sofia is correct. The missing number is 0.3.

Check your progress

1 a >   **b** <   **c** >

2 −3.6, −3.482, −3.449, −3.44, −3.06

3 a 0.08   **b** 0.21

4 2.6334

5 a 876   **b** 87.6   **c** 0.876
  d 365   **e** 3650   **f** 365

6 6

7 a

1	2	3	4	5	6	7	8	9
15	30	45	60	75	90	105	120	135

   b 25.47

   c $25.47 \times 1.5 = 38.205$

8 a 0.036 **b** 24.3 **c** 4.9

Unit 5

Getting started

1 a 55° **b** isosceles

2 a 141° **b** 58

3
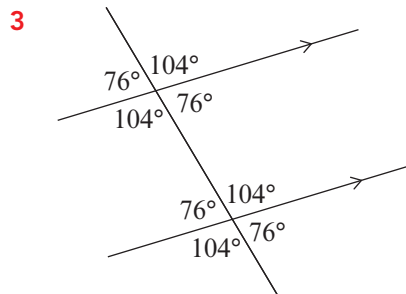
4 a learners' diagrams

   b 7.5 or 7.6 cm

Exercise 5.1

1 a p and t, q and u, s and w, r and v

   b q and w, r and t

2 a i b **ii** d

   b a and c or b and d

3 a q, r, u **b** p, s, t

4 a corresponding **b** alternate

   c CQX **d** BPY **e** YQD

5 $a = 136°$, alternate angles; $b = 136°$, corresponding angles or vertically opposite angles; $c = 44°$, angles on a straight line; $d = 44°$, alternate angles or angles on a straight line

6 If AB and CD were parallel, then the angles marked 50° and 40° would be equal. This is not the case.

7 a b, f, j

   b & c c and e; c and i

8 a neither **b** corresponding

   c corresponding **d** alternate

   e neither

9 a $h = b$ because they are <u>vertically opposite</u> angles; $b = d$ because they are <u>corresponding</u> angles; therefore $h = d$

   b There are several possible answers. Here is one: $h = b$ because they are vertically opposite angles; $b = f$ because they are alternate angles; $f = d$ because they are vertically opposite angles; therefore $h = d$

10 a They are alternate angles.

   b They are alternate angles.

   c $d + c + e = 180°$ because they are angles on a straight line. But $a = d$ and $b = e$ and so $a + c + b = 180°$

11 A labelled diagram, e.g.
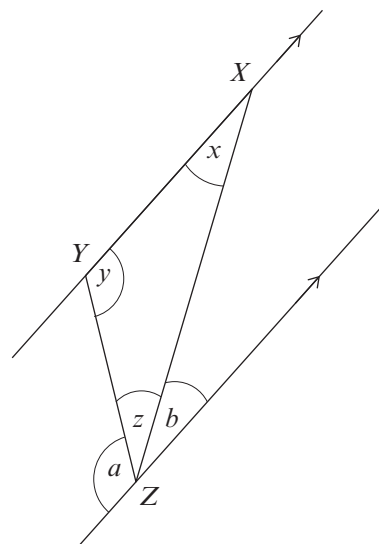
$a + z + b = 180°$ because they are angles on a straight line.

But $a = y$ because they are alternate angles

and $b = x$ because they are alternate angles

so therefore $y + z + x = 180°$

12 The easiest way is to label the angles like this:
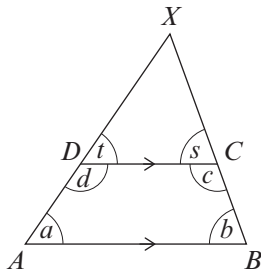
a $a = t$ because they are corresponding angles.
$b = s$ because they are corresponding angles.
The angle at X is common to both triangles.
This shows that the angles of the two triangles are the same.

b $d + t = 180°$ because they are angles on a straight line. But $t = a$ and so $d + a = 180°$.

c A similar proof shows that $b + c = 180°$

13 a Example answer:
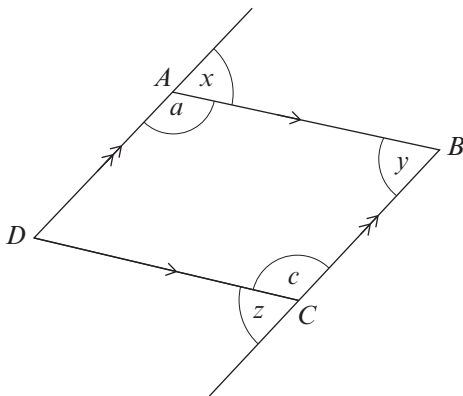
$a = 180 - x$ because they are angles on a straight line.

$x = y$ because they are alternate angles.

$y = z$ because they are corresponding angles.

$c = 180 - z$ because they are angles on a straight line.

So $c = 180 - z = 180 - x = a$

A similar argument shows that the angles at B and D are equal.

b learners' conclusions

Exercise 5.2

Because of the nature of proofs, alternative explanations are possible in some questions.

1 $a = 125°$ $b = 40°$ $c = 48°$

2 a $a = 137°$ $b = 113°$ $c = 110°$

b $55°$

3 a $68°$ and $72°$ **b** $140°$ and $112°$

4 a alternate angles

b alternate angles

c $x = a + y = a + c$

5 a alternate angles

b corresponding angles

c angle CBD = angle XDY, corresponding angles; angle BCD = angle CDX, alternate angles. The six angles round D add up to $360°$. The result follows from this.

6 Because $30°$ and $20°$ are opposite angles and must be equal / because $150°$ and $160°$ are opposite angles and must be equal.

7 a exterior angle of the triangle

b $e = a + b$; $f = b + c$

c $d + e + f = a + c + a + b + b + c = 2a + 2b + 2c = 2(a + b + c) = 2 \times 180°$ (angle sum of a triangle) $= 360°$

8 angle $BAC = 180 - (2 \times 68) = 44°$, isosceles triangle; angle $EDC = 44°$, corresponding angle

9 a Show that the angles of the triangle and the quadrilateral together make the angles of the pentagon. The sum of the angles is $180° + 360°$.

b learners' answers

10 a alternate angles

b $61°$

11 Angle $BAC = q$, alternate angles; $r = $ angle $BAC + p$, exterior angle. The result follows.
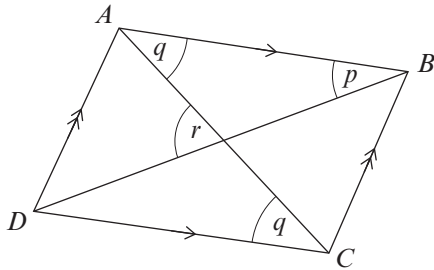
12 a $w = a + c$, exterior angle of a triangle; $y = b + d$, exterior angle of a triangle.

b $w + y = $ the sum of two angles of the quadrilateral; $x + z = $ the sum of the other two angles of the quadrilateral; $w + x + y + z = $ the angle sum of the quadrilateral $= 360°$

13 $a = 120°$; $b = 55°$; $c = 25°$

14 a exterior angle of a triangle

b exterior angle of a triangle

c $a + x + y = 180°$, angle sum of a triangle; hence $a + (b + d) + (c + e) = a + b + c + d + e = 180°$.

Exercise 5.3

For the constructions, accept answers for lengths within 0.2 cm, and answers for angles within 2°, to allow for slight drawing errors.

1 a construction

b $AC = 4.2$ cm and $BC = 5.7$ cm

2 a construction

b $XY = 7.1$ cm and $XZ = 4.2$ cm

3 a construction **b** 86°

4 a construction **b** 40°

5 a construction **b** 7.5 cm

c 37° and 53°

6 a construction **b** 77°

7 Sides of the constructed triangle should measure 5.8 cm, 7.8 cm and 7.1 cm

8 Diagram for Arun, with the hypotenuse of length 10.5 cm opposite a right angle, and diagram for Zara with the hypotenuse of length 13.4 cm opposite a right angle.

The triangles could look like this:
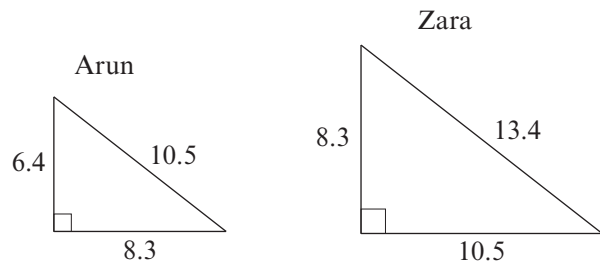
9 a and b construction **c** 6.4 cm

10 a $R = 109°$

b ST must be less than 11 cm (the sum of the two shorter sides), and must be more than 7.8 cm (Pythagoras)

11 a and b construction **c** 3.3 cm

12 a, b constructions to show Marcus's theory is correct

c, d constructions to show Sofia's theory is correct

Reflection

You can draw triangles of different sizes, with the same angles, because you are not given any lengths.

Check your progress

1 a i vertically opposite

ii corresponding **iii** alternate

b e and $j + k$ are alternate angles

c c and $i + j$ are corresponding angles

2 $A = 105°$, alternate angles; $B = 180° - 64° = 116°$, angles on a straight line; $C = 64°$, alternate angles; $D = 180° - 105° = 75°$, angles on a straight line. Other explanations are possible.

3 $x = 68°$ and $y = 58°$

4 Angle $B = 70°$ (corresponding angles). Angle $A = 180 - (40 + 70) = 70°$ (angle sum of a triangle). Two angles are equal so ABC is isosceles. Other explanations are possible.

5 a and b construction

c 2.3 cm and 2.8 cm

6 a and b construction

c 5.8 cm

Unit 6

Getting started

1 learners' examples

2 Possible answers are drawing three names out of a hat for girls and the same for boys; taking the first three boys and three girls on the register; taking the first three boys and three girls who walk into the room; and so on.

3 a It is more likely to be representative of the whole population.

b It takes more time to do and could cost more if you are paying people to do it.

4 A questionnaire and an interview are two possible answers.

Exercise 6.1

1 a do an experiment, drop a drawing pin many times; categorical data

b observation at the door of the shop on a number of days; discrete data

c a survey; discrete data

d a survey; continuous data

e counting the lengths for a number of sentences; discrete data

f a survey; continuous data

2 a gender; whether people usually come every week; whether they usually come at a particular time.

b it is all categorical data

c name is not required; gender is missing. 'How often?' is the wrong question. What they prefer is not the right question.

d This is a better data sheet. Circle the correct answer.

Gender	Do you usually come every week?	Do you usually come at the same time?
M F	Yes No	Yes No
M F	Yes No	Yes No

e You need people from all times of the day. It is better to choose a number of people from different times of the day, such as a number of people each hour.

3 a age, frequency of visits, category of film

b interview people or give them a questionnaire to complete

c They have all come to see a particular type of film so they will probably prefer that. It will not be representative of all the customers.

d The manager should ask people at different times when different films are being shown. Ask more people on a night when more people come.

e peer discussion

4 a time taken (continuous); gender (categorical); age (discrete)

b This is a possible sheet with one row for each child.

Time (seconds)	Gender	Age (years)
	M F	
	M F	

5 a the number of people in each car and the gender of the driver

b The first is discrete and the second is categorical.

c One possibility is a 2-way tally chart.

People	Female driver	Male driver
1		
2		
3		
4		
5		
6		

6 a Possible answers are the vocabulary, the size of the font, the number of pages, the lengths of the sentences, the lengths of the words.

b to f learners' own answers

Exercise 6.2

There are alternative answers to some questions. Examples given.

1 a i it is quick and easy

ii the first customers may not be representative

    b **i**   it will give a variety of customers

        ii it will take longer and it will be less convenient

    c choose 25 women and 25 men

    d age could be a factor

2 **a** There are many ways to do this.

    b–e learners' own answers

3 **a** advantage: it is easy and quick; disadvantage: it may not be typical of the whole day

    b Take a smaller number at different times of day. For example, 40 at 5 different times. This overcomes the disadvantage in part **a**.

4 learners' own answers

5 **a** No because 'too little' has the smallest frequency.

    b Yes, because nearly half think that there is too little homework.

    c One reason could be that the samples were chosen from parents of children in two different age groups and so they could have different opinions.

6 **a** It may not be representative of all the workers. For example, they could all come from the same department. It may only be people who have an early lunch.

    b A better method would take people at different times. You could also make sure you include men and women of different ages.

7 **a** Yes for Marcus and Zara because the mode is 1. No for Sofia because both 2 people and 3 people have a greater frequency / the mode is 2.

    b There are different numbers of cars in each survey. Marcus has most, Sofia least. The shape is the same for Marcus and Zara but different for Sofia.

    c Example answer: The samples were taken at different times of day and people were driving for different reasons. E.g. People going to work may often travel on their own, but may car share. People driving for social reasons could be on their own or in larger groups.

8 learners' own answers

Check your progress

1 This could include gender (categorical), age in years (discrete), height (continuous), number of successes in a particular number of throws (discrete).

2 You could get some data from a questionnaire or an interview. The data from throws could be observed or you could ask learners to tell you their success.

3 learners' choice and explanation

Unit 7

Getting started

1 **a** $\frac{5}{3} \neq 1\frac{1}{3}$ **b** $2\frac{1}{2} = \frac{10}{4}$ **c** $3\frac{5}{6} \neq \frac{21}{4}$

2 **a** $\frac{2}{3} < \frac{5}{3}$ **b** $2\frac{1}{2} > \frac{9}{4}$ **c** $\frac{2}{5} < \frac{3}{7}$

3 **a** $5\frac{2}{3}$ **b** $7\frac{5}{12}$

4 **a** $\frac{7}{12}$ **b** $\frac{16}{21}$

5 **a** 75   **b** 128   **c** 3800

Exercise 7.1

1 **a** $\frac{1}{2} = 0.5$ Terminating decimal

    b $\frac{1}{3} = 0.\dot{3}$ Recurring decimal

    c $\frac{1}{4} = 0.25$ Terminating decimal

    d $\frac{1}{5} = 0.2$ Terminating decimal

    e $\frac{1}{6} = 0.1\dot{6}$ Recurring decimal

    f $\frac{1}{7} = 0.\dot{1}42857\dot{7}$ Recurring decimal

    g $\frac{1}{8} = 0.125$ Terminating decimal

    h $\frac{1}{9} = 0.\dot{1}$ Recurring decimal

    i $\frac{1}{10} = 0.1$ Terminating decimal

    j $\frac{1}{11} = 0.\dot{0}\dot{9}$ Recurring decimal

    k $\frac{1}{12} = 0.08\dot{3}$ Recurring decimal

2 a

Unit fraction	$\frac{1}{2}$	$\frac{1}{3}$	$\frac{1}{4}$	$\frac{1}{5}$	$\frac{1}{6}$	$\frac{1}{7}$	$\frac{1}{8}$	$\frac{1}{9}$	$\frac{1}{10}$	$\frac{1}{11}$	$\frac{1}{12}$
Decimal	0.5	$0.\dot{3}$	0.25	0.2	$0.1\dot{6}$	$0.\dot{1}4285\dot{7}$	0.125	$0.\dot{1}$	0.1	$0.0\dot{9}$	$0.083\dot{3}$
Terminating (T) or recurring (R)	T	R	T	T	R	R	T	R	T	R	R

b i Zara is correct. $\frac{1}{16}=0.0625$ and $\frac{1}{32}=0.03125$.

Half of 0.5 is 0.25. From now on every halving means halving a decimal number with 25 on the end. Half of 25 is 12.5, so the final two digits of each fraction after 0.5 will always end in 25.

ii learners' answers

For example: All the fractions with a denominator which is a multiple of 3 are recurring decimals.

For example: The fractions with denominator 5, 10, 20, 40 (i.e. doubling each time) are terminating decimals. $\frac{1}{5}=0.2$, $\frac{1}{10}=0.1$, $\frac{1}{20}=0.05$, $\frac{1}{40}=0.025$, $\frac{1}{80}=0.0125$

iii peer discussion

3 a learners' answers

For example: Terminating, because all the denominators are factors of 100.

b A $\frac{5}{8}=0.625$, B $\frac{3}{4}=0.75$,

C $\frac{7}{10}=0.7$, D $\frac{11}{20}=0.55$,

E $\frac{3}{5}=0.6$

c D, E, A, C, B

4 a learners' answers

For example, recurring, because all the denominators are multiples of 3.

b A $\frac{5}{6}=0.8\dot{3}$, B $\frac{2}{3}=0.\dot{6}$,

C $\frac{7}{12}=0.58\dot{3}$, D $\frac{5}{9}=0.\dot{5}$,

E $\frac{3}{11}=0.\dot{2}\dot{7}$

c E, D, C, B, A

5 a learners' answers

For example: It's rounded the last 8 on the screen to a 9.

b $\frac{8}{9}$ | 0.888888889 | $\frac{1}{9}$ | 0.111111111 |

$\frac{11}{15}$ | 0.733333333 | $\frac{7}{18}$ | 0.388888889 |

c learners' answers

For example: Changes the fraction to a decimal.

learners' answers

For example: Changes the decimal back to a fraction.

d i $\frac{7}{15}=0.4\dot{6}$ **ii** $\frac{8}{11}=0.\dot{7}\dot{2}$

6 a $\frac{7}{9}=0.\dot{7}$ **b** $\frac{13}{20}=0.65$

c $\frac{2}{15}=0.1\dot{3}$ **d** $\frac{9}{40}=0.225$

7 learners' answers

For example: The last two the digits are the same as the first two, so it must be repeating.

8 a $\frac{2}{7}=0.\dot{2}8571\dot{4}$ **b** $\frac{9}{13}=0.\dot{6}9230\dot{7}$

c $\frac{11}{14}=0.7\dot{8}5714\dot{2}$

9 a, b i $\frac{5}{12}=0.41\dot{6}$ is correct

ii $\frac{10}{11}=0.\dot{9}0$ is wrong as the recurring dot should be over the 9 and the 0, so $0.\dot{9}\dot{0}$

iii $\frac{6}{7}=0.8\dot{5}7142\dot{2}$ is wrong as the recurring dot should be over the 8 at the start, not the 5, so $0.\dot{8}5714\dot{2}$

iv $\frac{1}{37}=0.0\dot{2}\dot{7}$ is wrong as the recurring dot should be over the 0 at the start, not the 2, so $0.\dot{0}2\dot{7}$

10 a $\frac{4}{3}=1.\dot{3}$ **b** $\frac{13}{6}=2.1\dot{6}$

c $\frac{19}{9}=2.\dot{1}$ **d** $\frac{45}{11}=4.\dot{0}\dot{9}$

11 a $4.\dot{3}$ **b** $1.\dot{6}$

    c $6.1\dot{6}$ **d** $3.8\dot{3}$

12 $\frac{8}{52} = 0.\dot{1}5384\dot{6}$

13 Yes. Both $\frac{1}{15}$ and $\frac{4}{15}$ have one number that is recurring and both $\frac{1}{22}$ and $\frac{7}{22}$ have two recurring decimals.

Exercise 7.2

1 a $\frac{11}{4} \neq 2\frac{16}{20}$ **b** $\frac{45}{6} = 7\frac{1}{2}$

    c $-\frac{15}{8} \neq -2\frac{1}{8}$ **d** $-8\frac{4}{5} = -\frac{132}{15}$

2 a $\frac{13}{2} < 6\frac{5}{8}$ **b** $\frac{17}{3} < 6\frac{7}{12}$

    c $5\frac{3}{5} > \frac{82}{15}$ **d** $\frac{19}{4} < 4\frac{4}{5}$

    e $-\frac{17}{4} > -4\frac{5}{12}$ **f** $-\frac{7}{3} > -2\frac{5}{9}$

    g $-\frac{21}{5} < -4\frac{2}{15}$ **h** $-\frac{8}{5} > -1\frac{5}{7}$

3 learners' answers

4 a $-\frac{7}{4}$ **b** $-\frac{83}{20}$ **c** $-\frac{37}{6}$

5 a **i** $\frac{3}{5}$ **ii** $\frac{7}{9}$ **iii** $\frac{19}{11}$

    b When the denominators are the same, the larger the numerator the larger the fraction.

    c **i** $\frac{1}{5}$ **ii** $\frac{2}{3}$ **iii** $\frac{13}{4}$

    d When the numerators are the same, the larger the denominator the smaller the fraction.

6 a $\frac{3}{11} < \frac{5}{11}$ **b** $\frac{7}{18} > \frac{5}{18}$ **c** $\frac{12}{7} > \frac{10}{7}$

    d $\frac{8}{17} > \frac{8}{19}$ **e** $\frac{9}{13} < \frac{9}{10}$ **f** $\frac{15}{4} > \frac{15}{7}$

7 $-3\frac{1}{4}, -\frac{13}{6}, -\frac{17}{8}, -\frac{7}{13}$ and learners' answers

8 $-4\frac{2}{5}, -\frac{19}{5}, -3\frac{1}{6}, -\frac{9}{11}$

9 Belle [Adele scored $\frac{16}{25} = 64\%$, Belle scored $\frac{13}{20} = 65\%$, and Catrina scored 63%]

10 Irena [Steffan has $\frac{34}{40} = 85\%$, Irena has 87%]

11 a **i** $\frac{4}{7} = 0.5714\ldots$ $-1\frac{4}{7} = -1.5714\ldots$

      ii $\frac{5}{9} = 0.5555\ldots$ $-1\frac{5}{9} = -1.5555\ldots$

      iii $\frac{7}{12} = 0.5833\ldots$ $-1\frac{7}{12} = -1.5833\ldots$

    b $-\frac{19}{12}, -\frac{11}{7}, -\frac{14}{9}$

12 a $-\frac{37}{9} = -4.11\ldots, -\frac{25}{6} = -4.16\ldots,$

      $-\frac{209}{50} = -4.18, -\frac{47}{11} = -4.27\ldots$

    b $-\frac{47}{11}, -\frac{209}{50}, -\frac{25}{6}, -\frac{37}{9}$

13 $-\frac{82}{15}, -5\frac{3}{8}, -\frac{107}{20}, -\frac{37}{7}$

14 The first day = 92% [Second day = 90.3 . . . %]

15 a First: $\frac{1235}{1368}$ Second: $\frac{1260}{1368}$

    b First: $0.902\dot{7}$ Second: $0.921\ldots$

    c learners' answers

    d Second

16 Group A [group A, $0.891\ldots$, group B, $0.859\ldots$]

17 First card: any two fractions such that $-\frac{180}{63} < x < -\frac{175}{63}$

    Second card: any two fractions such that $-\frac{125}{45} < y < -\frac{117}{45}$

Exercise 7.3

1 a $5\frac{1}{3} - 2\frac{2}{3}$ ① $\frac{16}{3} - \frac{8}{3}$ ② $\frac{16}{3} - \frac{8}{3} = \frac{8}{3}$ ③ $\frac{8}{3} = 2\frac{2}{3}$

    b $9\frac{1}{6} - 3\frac{5}{12}$ ① $\frac{55}{6} - \frac{41}{12}$ ② $\frac{55}{6} - \frac{41}{12} = \frac{110}{12} - \frac{41}{12} = \frac{69}{12}$

      ③ $\frac{69}{12} = \frac{23}{4} = 5\frac{3}{4}$

    c $5\frac{3}{4} - 3\frac{5}{6}$ ① $\frac{23}{4} - \frac{23}{6}$ ② $\frac{23}{4} - \frac{23}{6} = \frac{69}{12} - \frac{46}{12} = \frac{23}{12}$

      ③ $\frac{23}{12} = 1\frac{11}{12}$

    d $4\frac{1}{4} - 1\frac{3}{5}$ ① $\frac{17}{4} - \frac{8}{5}$ ② $\frac{17}{4} - \frac{8}{5} = \frac{85}{20} - \frac{32}{20} = \frac{53}{20}$

      ③ $\frac{53}{20} = 2\frac{13}{20}$

2 a $\frac{3}{4}$ **b** $1\frac{9}{10}$ **c** $2\frac{3}{4}$ **d** $2\frac{5}{12}$

3 learners' answers

4 a $1\frac{9}{14}$ **b** $4\frac{3}{4}$ **c** $4\frac{5}{12}$ **d** $1\frac{23}{36}$

5 a Yes. $9 - 3 = 6$, then the answer could be within $6 + 1$ and $6 - 1$.

    b learners' answers

    c Subtract the whole numbers then work out $+1$ and -1 from this answer.

    d Subtract the whole numbers then work out:

      (for 3 mixed numbers) $+2$ and -2 from this answer

(for 4 mixed numbers) +3 and −3 from this answer

(for 5 mixed numbers) +4 and −4 from this answer, etc.

6 a Answer between 0 and 2   **b** $\frac{5}{8}$

7 $\frac{3}{4}$ m

8 a Answer between 3 and 5   **b** $4\frac{1}{12}$

9 learners' answers

10 $5\frac{9}{40}$ km

11 $2\frac{13}{30}$ kg

12 a learners' answers

For example: He has worked out $4\frac{9}{10} - \frac{6}{10}$

b $3\frac{7}{10}$

13

		$12\frac{3}{4}$		
	$8\frac{2}{3}$		$4\frac{1}{12}$	
$5\frac{13}{15}$		$2\frac{4}{5}$		$1\frac{17}{60}$
$4\frac{14}{45}$	$1\frac{5}{9}$		$1\frac{11}{45}$	$\frac{7}{180}$
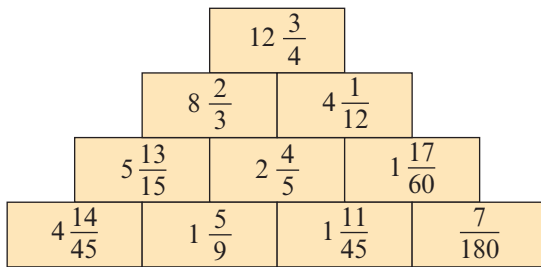
14 $11\frac{3}{4}$ m

Exercise 7.4

1 a $3\frac{1}{2} \times 8 = 3 \times 8 + \frac{1}{2} \times 8$
$= 24 + 4$
$= 28$

b $2\frac{1}{4} \times 12 = 2 \times 12 + \frac{1}{4} \times 12$
$= 24 + 3$
$= 27$

c $4\frac{2}{3} \times 9 = 4 \times 9 + \frac{2}{3} \times 9$
$= 36 + 6$
$= 42$

d $8\frac{3}{5} \times 10 = 8 \times 10 + \frac{3}{5} \times 10$
$= 80 + 6$
$= 86$

2 a $15 \times 2 = 30 \, \text{m}^2$ **b** $35 \, \text{m}^2$

3 No, it's 48 litres not 46 litres.

4 a $4\frac{1}{2} \times 9 = 4 \times 9 + \frac{1}{2} \times 9$
$= 36 + \frac{9}{2}$
$= 36 + 4\frac{1}{2}$
$= 40\frac{1}{2}$
estimate: 5 × 9 = 45

b $3\frac{3}{4} \times 11 = 3 \times 11 + \frac{3}{4} \times 11$
$= 33 + \frac{33}{4}$
$= 33 + 8\frac{1}{4}$
$= 41\frac{1}{4}$
estimate: 4 × 11 = 44

c $5\frac{2}{3} \times 7 = 5 \times 7 + \frac{2}{3} \times 7$
$= 35 + \frac{14}{3}$
$= 35 + 4\frac{2}{3}$
$= 39\frac{2}{3}$
estimate: 6 × 7 = 42

d $2\frac{2}{5} \times 6 = 2 \times 6 + \frac{2}{5} \times 6$
$= 12 + \frac{12}{5}$
$= 12 + 2\frac{2}{5}$
$= 14\frac{2}{5}$
estimate: 2 × 6 = 12

5 a $5 \times (12 + 5) = 85 \, \text{cm}^2$ **b** $87\frac{2}{9} \, \text{cm}^2$

6 a learners' answers   **b** $7\frac{1}{5} \, \text{m}^2$

c No, they will cost $336. She needs to round $7\frac{1}{5}$ m² up to 8 m² then 8 × 42 = $336. She has rounded $7\frac{1}{5}$ m² down to 7 m² then 7 × 42 = $294.

7 a learners' answers

b i $14\frac{4}{7}$ **ii** $78\frac{2}{3}$

c learners' answers

8 a learners' answers

b i $33\frac{3}{4}$ **ii** $66\frac{1}{2}$ **iii** $32\frac{2}{5}$

9 $262\frac{1}{2}$ minutes = 4 hours $22\frac{1}{2}$ minutes

10 a i $6\frac{2}{5}$ **ii** $9\frac{3}{5}$

b $3\frac{1}{5} \times 5 = $ a whole number answer

c $3\frac{2}{5} \times 5 =$ a whole number answer

$3\frac{3}{5} \times 5 =$ a whole number answer

$3\frac{4}{5} \times 5 =$ a whole number answer

d They are the same.

e $3\frac{1}{7}, 3\frac{2}{7}, 3\frac{3}{7}, 3\frac{4}{7}, 3\frac{5}{7}$ and $3\frac{6}{7}$, all $\times 7 =$ a whole number answer

f learners' answers

11 a $2 \times (13 - 5) = 16\,\text{m}^2$

b $15\frac{13}{15}\,\text{m}^2$

Exercise 7.5

1 a 6   **b** 8   **c** 12   **d** 25

2 a, b learners' answers

   c i 36   **ii** 100   **iii** 72

3 80 m

4 a 120 km/h

   b 90 km/h

   c 96 km/h

5 peer discussion

6 a 6   **b** 8   **c** 10   **d** 14

7 a learners' answers

   b $5\frac{1}{3}$

   c $4 \div \frac{3}{4} = 4 \times \frac{4}{3} = \frac{16}{3} = 5\frac{1}{3}$

   d peer discussion

   e peer discussion

8 a $11 \div \frac{3}{4} = 11 \times \frac{4}{3} = \frac{44}{3} = 14\frac{2}{3}$

   b $9 \div \frac{5}{6} = 9 \times \frac{6}{5} = \frac{54}{5} = 10\frac{4}{5}$

   c $8\frac{3}{4}$ **d** $17\frac{1}{7}$ **e** $27\frac{1}{2}$

9 a learners' answers

   b i $10\frac{1}{2}$ **ii** $7\frac{1}{3}$

     iii $13\frac{1}{3}$ **iv** $9\frac{3}{4}$

10 ① is correct. Learners' answers. For example, when you divide an integer by 1, you get the same integer. When you divide an integer by a fraction smaller than 1 you are finding how many smaller parts go into that integer, so the answer is going to be more than the integer.

② is incorrect. Learners' answers. For example, a bigger fraction will go into an integer fewer times than a smaller fraction.

11 a i 6, 12, 18, 24, . . .

     ii 30, 36

     iii 6 times table

   b i 3, 6, 9, 12, . . .

     ii 15, 18

     iii 3 times table

   c learners' answers

     For example, the terms in the sequence in **b** are half the terms in the sequence in **a**. This is because when you divide by a fraction that is doubled, all your answers are halved.

   d 2, 4, 6, 8, . . .

     learners' answers

     For example, divide the terms in the sequence in **a** by 3.

   e 3, 6, 9, 12, . . .

     learners' answers

     For example, divide the terms in the first sequence by 5.

Exercise 7.6

1 a $\frac{1}{2}$ **b** $\frac{3}{8}$ **c** $\frac{7}{10}$ **d** $\frac{1}{2}$

2 a $\frac{2}{3}$ **b** $\frac{7}{8}$ **c** $\frac{7}{10}$ **d** $\frac{7}{8}$

   e $1\frac{1}{6}$ **f** $1\frac{4}{15}$ **g** $\frac{2}{9}$ **h** $\frac{1}{8}$

   i $\frac{2}{15}$ **j** $\frac{1}{2}$ **k** $\frac{7}{10}$ **l** $\frac{3}{20}$

3 a $\frac{8}{15}$ **b** $\frac{11}{28}$ **c** $\frac{19}{45}$ **d** $1\frac{5}{12}$

   e $\frac{33}{40}$ **f** $1\frac{1}{12}$ **g** $\frac{1}{6}$ **h** $\frac{11}{20}$

   i $\frac{3}{14}$ **j** $\frac{13}{28}$ **k** $\frac{5}{24}$ **l** $\frac{5}{36}$

4 a $\frac{3}{10}$

   b learners' answers

5 $\frac{5}{12}$

6 $\frac{13}{30}$

7 a 6   **b** 10   **c** 18

   d 15   **e** 12   **f** 12

8 $18 \div \frac{5}{7} = \frac{126}{5} = 25\frac{1}{5}$

9 a i $11\frac{1}{4}$ **ii** $11\frac{2}{3}$

    iii $16\frac{1}{2}$ **iv** $11\frac{1}{5}$

   b, c learners' answers

10 $13\frac{1}{3}$ m

11 a learners' answers

For example, when working out the subtraction he has not found a common denominator but just subtracted the numerators and subtracted the denominators.

   b $\frac{1}{9}$

12 a 4   **b** 6   **c** $\frac{2}{3}$

   d $\frac{1}{12}$ **e** $\frac{1}{4}$ **f** 20

13 1st sequence is: B $1\frac{1}{2}$, D 3, A $4\frac{1}{2}$, C 6

First term is $1\frac{1}{2}$. Term to term rule is 'add $1\frac{1}{2}$'.

2nd sequence is: C 6, A $4\frac{1}{2}$, D 3, B $1\frac{1}{2}$

First term is 6. Term to term rule is 'subtract $1\frac{1}{2}$'.

Check your progress

1 a $\frac{3}{8} = 0.375$ terminating

   b $\frac{4}{9} = 0.\dot{4}$ recurring

2 $-\frac{17}{6}, -\frac{27}{10}, -\frac{38}{15}, -\frac{12}{5}$

3 a $3\frac{11}{12}$ **b** $43\frac{1}{2}$

4 a 24   **b** $13\frac{1}{3}$

5 a $\frac{7}{10}$ **b** $\frac{4}{21}$

   c 14   **d** $3\frac{3}{4}$

Unit 8

Getting started

1 i a 2   **b** 2

   ii a 1   **b** 1

   iii a 0   **b** 1

   iv a 8   **b** 8

2 a i 6   **ii** 12   **iii** 8

   b

Top view

Front view   Side view
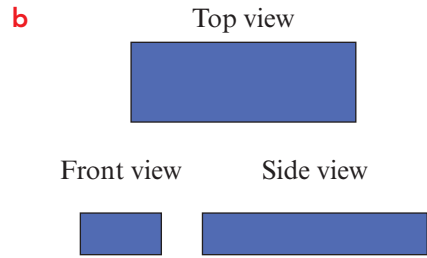
3 a $h = x + p - t$ **b** $h = 4x$

   c $h = \frac{y}{3x}$

4
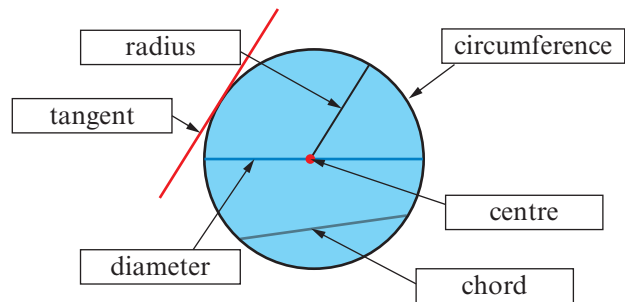
5 a cube   **b** cuboid

   c cylinder   **d** cone

   e sphere   **f** tetrahedron

   g square-based pyramid

   h triangular prism

6 a 5 m   **b** 60 cm

Exercise 8.1

1 a

Name of regular polygon	Number of sides	Number of lines of symmetry	Order of rotational symmetry
Pentagon	5	5	5
Hexagon	6	6	6
Heptagon	7	7	7
Octagon	8	8	8
Nonagon	9	9	9
Decagon	10	10	10

   b They are all equal.

c The number of sides of a regular polygon is the same as/equal to the number of lines of symmetry.

The number of sides of a regular polygon is the same as/equal to the order of rotational symmetry.

d **i** 11   **ii** 12

2 **a** True

b False. AB is parallel to CD

c False. BD is parallel to AC

d True

3 **a** A square has:
- all sides the same length
- two pairs of parallel sides
- all angles are 90°.

b A rectangle has:
- two pairs of sides the same length
- two pairs of parallel sides
- all angles are 90°.

c A rhombus has:
- all sides the same length
- two pairs of parallel sides
- opposite angles are equal.

d A parallelogram has:
- two pairs of sides the same length
- two pairs of parallel sides
- opposite angles are equal.

4 peer discussions.

a A square is a special rectangle.

b A square is a special rhombus.

c A rectangle is a special parallelogram.

d A rhombus is a special parallelogram.

5 learners' answers

Example: No she hasn't. She could be describing either a rectangle or a square. She also needs to say that all the sides have the same length.

6 **a** False. AC is the same length as BD

b True

c False. Angle CAB is the same size as angle ABD

d True

7 **a** A trapezium has:
- one pair of parallel sides.

b An isosceles trapezium has:
- one pair of sides the same length
- one pair of parallel sides
- two pairs of equal angles.

c A kite has:
- two pairs of sides the same length
- one pair of equal angles.

8 peer discussions.

a An isosceles trapezium is always a trapezium but a trapezium is not always an isosceles trapezium.

b A rhombus is a special kite.

c A parallelogram is a special trapezium.

9 learners' answers

Example: No, he hasn't. He has said that the kite has two pairs of equal angles, but it only has one pair of equal angles.

10 **a** J   **b** K   **c** N   **d** L

e I   **f** M   **g** H

11 **a** True   **b** False   **c** True

Exercise 8.2

1 **a** $C = \pi d$
$= 3.14 \times 6$
$= 18.84$
$= 18.8\,\text{cm}\ (1\,\text{d.p.})$

b $C = \pi d$
$= 3.14 \times 25$
$= 78.5$
$= 78.5\,\text{cm}\ (1\,\text{d.p.})$

c $C = \pi d$
$= 3.14 \times 4.25$
$= 13.345$
$= 13.3\,\text{m}\ (1\,\text{d.p.})$

2 **a** $d = 2 \times r$
$= 2 \times 7$
$= 14\,\text{cm}$
$C = \pi d$
$= 3.142 \times 14$
$= 43.988$
$= 43.99\,\text{cm}\ (2\,\text{d.p.})$

b $d = 2 \times r$
$= 2 \times 2.6$
$= 5.2\,\text{cm}$
$C = \pi d$
$= 3.142 \times 5.2$
$= 16.3384$
$= 16.34\,\text{cm}\ (2\,\text{d.p.})$

c $d = 2 \times r$

    $= 2 \times 0.9$

    $= 1.8\,\text{m}$

  $C = \pi d$

    $= 3.142 \times 1.8$

    $= 5.6556$

    $= 5.66\,\text{m}$ (2 d.p.)

3 a $37.699\,111\,84\ldots$ cm

  b i 37.68 cm

    ii 37.704 cm

    iii $37.714\,285\,71\ldots$ cm

  c 3.142

  d learners' answers and explanations
Example: π button gives the most accurate answer, but if you don't have a calculator then 3.14 is easiest to use.

4 a 28.27 cm   **b** 22.78 m

  c 69.12 cm   **d** 20.11 m

5 learners' formulae – any version of $C = 2\pi r$

6 a learners' answers

  b learners' answers and explanations

  c i 8.91 cm (3 s.f.)

    ii 1.46 m (3 s.f.)

  d $r = \dfrac{C}{2\pi}$

  e i 2.39 cm (3 s.f.)

    ii 1.47 m (3 s.f.)

7 12.4 cm or 124 mm

8 $r = \dfrac{C}{2\pi} = \dfrac{5.65}{2 \times \pi} = 0.899225\ldots = 0.9\,\text{cm} = 9\,\text{mm}$

9 a i 51.42 cm   **ii** 38.56 m

    iii 41.13 cm   **iv** 33.42 m

  b learners' answers and explanations

  Example for the perimeter of a quarter-circle is: $P = \dfrac{\pi d}{4} + 2r$ or $P = \dfrac{\pi d}{4} + d$ or $P = \dfrac{\pi r}{2} + 2r$

10 Semicircle: $P = \dfrac{\pi \times 15}{2} + 15 = 38.56\,\text{m}$

  Quarter-circle: $P = \dfrac{\pi \times 10}{2} + 2 \times 10 = 35.71\,\text{m}$

  Zara is correct as 38.56 m > 35.71 m

11 a 41.25 cm   **b** 25.07 m

  c 159.95 mm   **d** 20.31 cm

Exercise 8.3

1 a

3D shape	Number of faces	Number of vertices	Number of edges
cube	6	8	12
cuboid	6	8	12
tetrahedron	4	4	6
square-based pyramid	5	5	8
triangular prism	5	6	9
trapezoidal prism	6	8	12

  b number of faces + number of vertices is two more than the number of edges

    or any rearrangement of this, e.g. number of faces + number of vertices − number of edges = 2

  c $E = F + V - 2$ **or** any rearrangement of this, e.g. $F + V = E + 2$

  d peer discussions

  e The formula only works for shapes with flat faces.

    learners' explanations

    Example: a cylinder has 1 curved surface, 0 vertices and 2 edges.

    If you use the formula $F + V = E + 2$ then $F + V = 1 + 0 = 1$ and $E + 2 = 2 + 2 = 4$, and $1 \neq 4$

2 a $6 \div 2 = 3\,\text{cm}$ **b** $8 \div 2 = 4\,\text{cm}$

                                            $3 \div 2 = 1.5\,\text{cm}$

                                            $5 \div 2 = 2.5\,\text{cm}$

  c $7 \div 2 = 3.5\,\text{cm}$

    $10 \div 2 = 5\,\text{cm}$

    $14 \div 2 = 7\,\text{cm}$

    Learners' scale drawings should be the following shapes with dimensions shown.

    The dimensions for all rectangles are given as width by height.

a Top view: Square of side 3 cm, Front view: Square of side 3 cm, Side view: Square of side 3 cm

b Top view: Rectangle 4 cm by 2.5 cm, Front view: Rectangle 4 cm by 1.5 cm, Side view: Rectangle 2.5 cm by 1.5 cm

c Top view: Circle radius 3.5 cm, Front view: Rectangle 7 cm by 5 cm, Side view: Rectangle 7 cm by 5 cm

3 a Seb is correct as you would see the extra edge that he has shown.

b Yes, they have used a scale of 1 : 1

c peer discussions

d accurate drawing of:
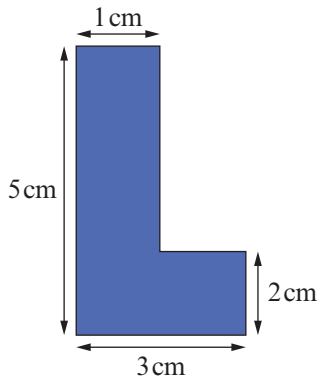
e accurate drawings of:

i

ii
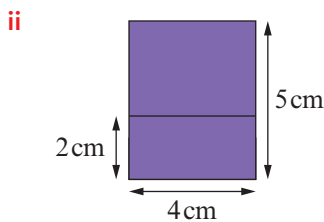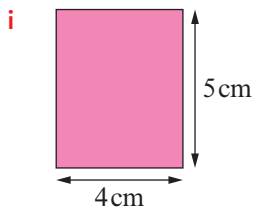
The drawings are not the same.

f peer discussions

4 accurate drawings of these:
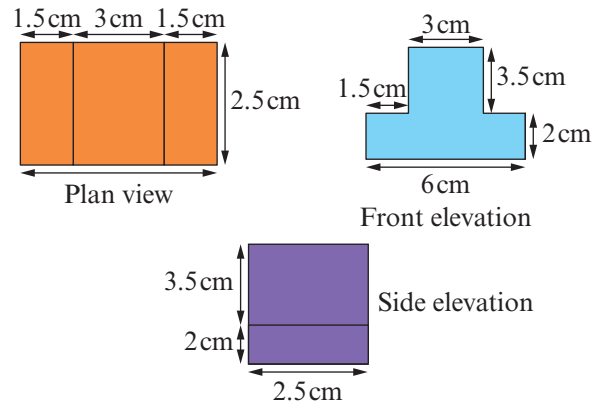
5 accurate drawings of these:
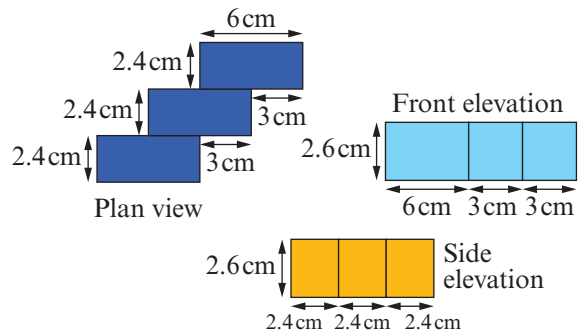
6 a Yes, you will see a rectangle 6 cm wide and 4 cm high.

b accurate drawings of these:
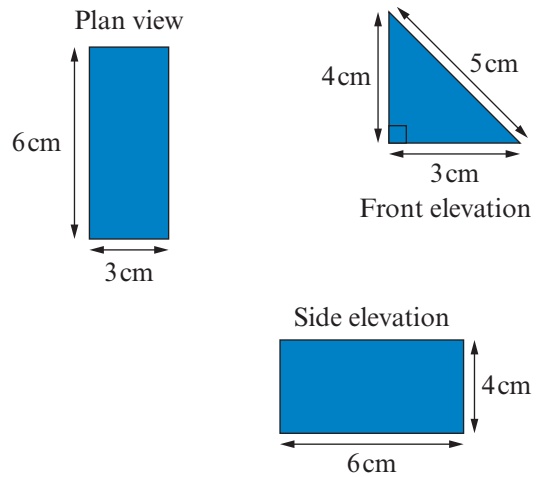
c Yes, you will see a rectangle 18 cm wide and 16 cm high.

d accurate drawings of these:

Plan view

4.5 cm

6 cm

5 cm

4 cm

6 cm

Front elevation

4 cm

4.5 cm

Side elevation

7 accurate drawings of these:

Plan view

10 cm

6 cm

Front elevation

1.5 cm

2 cm

6 cm

Side elevation

1.5 cm

2 cm

10 cm
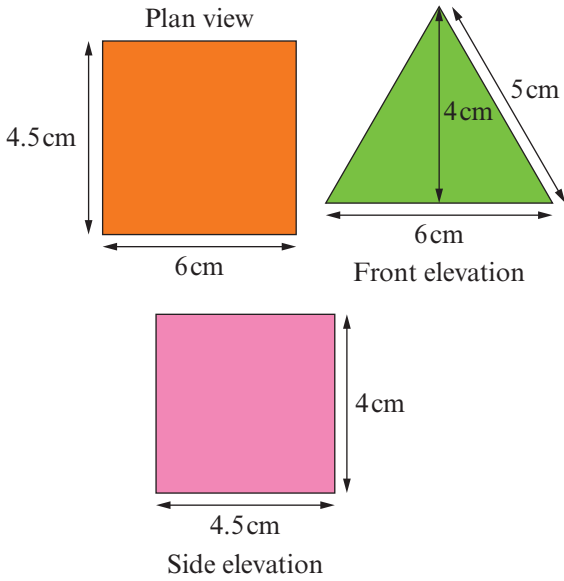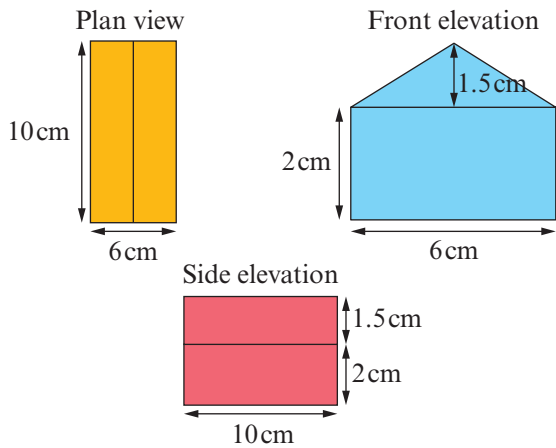
8 a i B   **ii** C   **iii** A

   b Yes, any shape which has the same base, of 6 squares in this arrangement.

Plan view
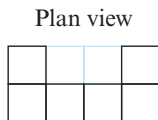
They can have any number of blocks built above them and the plan will remain the same.

   c No, the three views give you all the information you need to fix the shape.

9

	Plan view	Front elevation	Side elevation
a			
b			
c			
d			
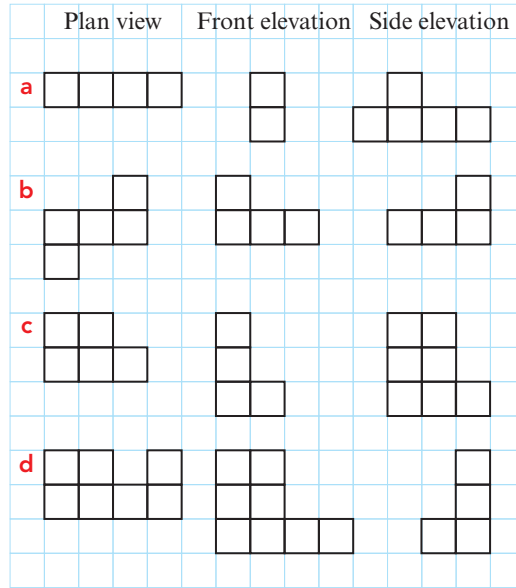
10 The side elevation is incorrect. He has drawn a rectangle 3 cm wide and 6 cm high, but it should be a rectangle 2 cm wide and 6 cm high.

11

8 cm

Plan view

2 cm

8 cm

Side elevation

2 cm

4 cm

2 cm

Front elevation

6 cm

12

Plan view	Front elevation	Side elevation
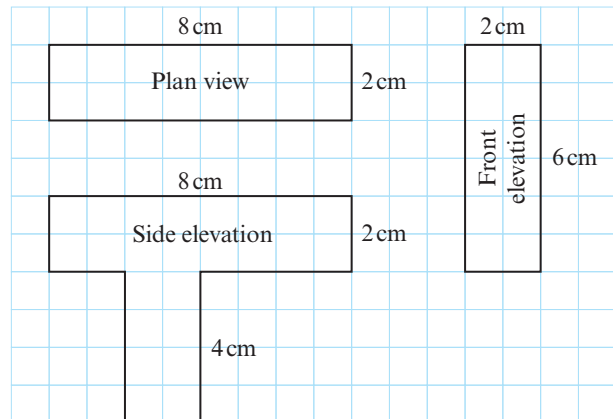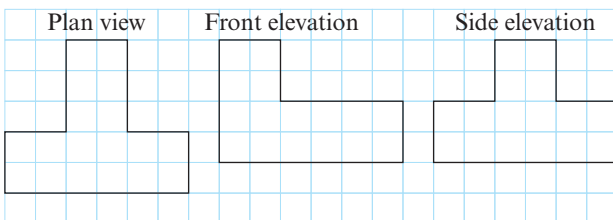
Check your progress

1 A regular pentagon has 5 sides of equal length. It has 5 lines of symmetry and rotational symmetry of order 5.

2 a True   **b** False

   c True   **d** True

3 **a** 40.84 cm **b** 16.96 m

4 8.6 cm or 86 mm

5 **a**
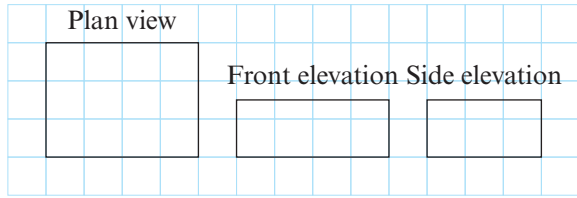
b
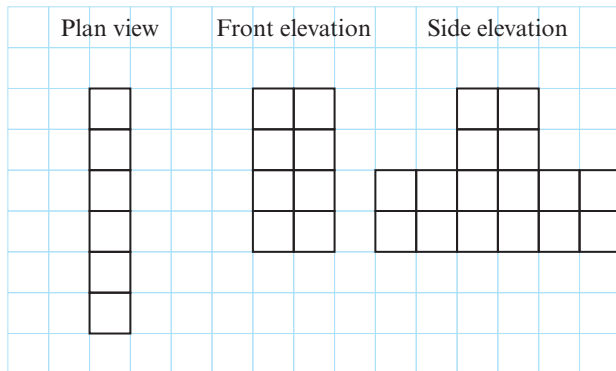
Unit 9

Getting started

1 **a** **i** add 3 **ii** 16, 19

  b **i** subtract 2 **ii** 20, 18

2 3, 6, 12, 24

3 **a** Pattern 4

  b
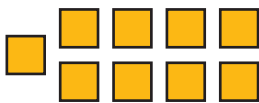
Pattern number	1	2	3	4	5
Number of squares	3	5	7	9	11

  c add 2 **d** 21

4 **a** 6, 12, 18, 24 **b** 0, 1, 2, 3

5 **a** Input 12, Outputs 6, 9

  b Input 12, Outputs 0, 8

  c Input 3, Outputs 40, 50

  d Inputs 24, 36, Output 2

Exercise 9.1

1 **a** The term-to-term rule is: add $3\frac{1}{2}$

  The next two terms are: 21, $24\frac{1}{2}$

  b The term-to-term rule is: subtract 0.2

  The next two terms are: 9.2, 9

2 **a** **i** add $\frac{1}{4}$ **ii** $6, 6\frac{1}{4}$

  b **i** add $1\frac{1}{3}$ **ii** $12\frac{2}{3}$, 14

  c **i** add 0.4 **ii** 7, 7.4

  d **i** subtract $\frac{1}{2}$ **ii** $7, 6\frac{1}{2}$

  e **i** subtract $\frac{2}{5}$ **ii** $8\frac{2}{5}$, 8

  f **i** subtract 0.8 **ii** 13.8, 13

3 **a** 1, 2.4, 3.8 **b** $6, 10\frac{1}{2}$, 15

  c 20, 17.5, 15 **d** $40, 34\frac{2}{3}, 29\frac{1}{3}$

  e 0.4, 0.8, 1.6 **f** 9, 4.5, 2.25

4 **a** $2, 4\frac{1}{5}, 6\frac{2}{5}, 8\frac{3}{5}, 10\frac{4}{5}, 13, 15\frac{1}{5}$

  b $5, 8\frac{3}{7}, 11\frac{6}{7}, 15\frac{2}{7}, 18\frac{5}{7}, 22\frac{1}{7}, 25\frac{4}{7}$

  c $25, 24\frac{3}{4}, 24\frac{1}{2}, 24\frac{1}{4}, 24, 23\frac{3}{4}, 23\frac{1}{2}$

  d $100, 89\frac{1}{2}, 79, 68\frac{1}{2}, 58, 47\frac{1}{2}, 37$

  e 8, 8.3, 8.6, 8.9, 9.2, 9.5, 9.8

  f 24.8, 24.4, 24, 23.6, 23.2, 22.8, 22.4

5 learners' explanations

  a 10.4 **b** Yes **c** No

6 **a** 8, 11, 17

  b 15, 18, 27

  c 12, 11, 10.5

7 **a** C subtract 3 then multiply by 2

  b 70

8 Yes, terms are 10, 8, 4, −4, . . .

9 **a** 4, 2, −4

  b 10, 15, 25

  c −6, 2, 6

10 a 5, 5, 5 The terms are all the same.

b learners' answers

e.g. first term 8, term-to-term rule is divide by 2 then add 4

e.g. first term 2, term-to-term rule is multiply by 7 then subtract 12

11 She is correct. The number she divides by 2 never reaches 20. Once she halves this number the terms in her sequence will never reach 10.

12 a You need to add another 10 lots of $4\frac{3}{5}$ to the 10th term, not double the 10th term.

b $96\frac{2}{5}$

13 50

Exercise 9.2

1 a 3, 5, 7, . . .   **b** add 2

c
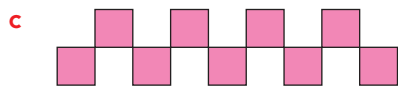
d Multiples of 2 squares are added to the 1st square

e Position-to-term rule is: term $= 2 \times$ position number $+ 1$

position number	1	2	3	4
term	3	5	7	9
2 × position number	2	4	6	8
2 × position number + 1	3	5	7	9

2 a 6, 10, 14, . . .   **b** add 4

c
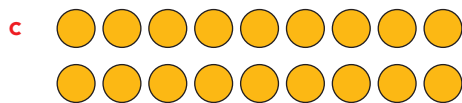
d Multiples of 4 dots are added to the first 2 dots

e Position-to-term rule is: term $= 4 \times$ position number $+ 2$

position number	1	2	3	4
term	6	10	14	18
4 × position number	4	8	12	16
4 × position number + 2	6	10	14	18

3 a 3, 8, 13, . . .

b add 5

c
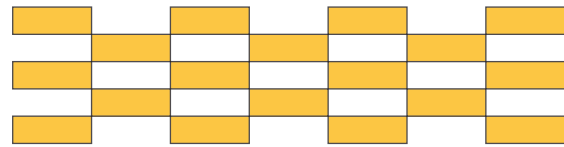
d Position-to-term rule is: term $= 5 \times$ position number $- 2$

position number	1	2	3	4
term	3	8	13	18
5 × position number	5	10	15	20
5 × position number − 2	3	8	13	18

4 Razi is wrong. The position-to-term rule for the sequence is: term $= 3 \times$ position number $+ 2$

5 a As the sequence increases by 2 every time, the position-to-term rule starts with term $= 2 \times$ position number $+ . . .$

b term $= 2 \times$ position number $+ 2$

6 a Position-to-term rule is: term $= 5 \times$ position number $+ 5$

b Position-to-term rule is: term $= 20 \times$ position number $- 10$

7 Pattern 20 will have 59 hexagons

peer discussions of the different methods

Position-to-term rule is: term $= 3 \times$ position number $- 1$

8 Pattern 18 will have 69 trapezia

Position-to-term rule is: term $= 4 \times$ position number $- 3$

Exercise 9.3

1 a nth term is $2n + 1$

1st term $= 2 \times 1 + 1$ 2nd term $= 2 \times 2 + 1$
$= 3$ $= 5$

3rd term $= 2 \times 3 + 1$ 4th term $= 2 \times 4 + 1$
$= 7$ $= 9$

b nth term is $3n - 2$

1st term $= 3 \times 1 - 2$ 2nd term $= 3 \times 2 - 2$
$= 1$ $= 4$

3rd term $= 3 \times 3 - 2$ 4th term $= 3 \times 4 - 2$
$= 7$ $= 10$

2 a $7, 8, 9, \ldots, 16$ **b** $-2, -1, 0, \ldots, 7$

c $9, 18, 27, \ldots, 90$ **d** $6, 12, 18, \ldots, 60$

e $7, 9, 11, \ldots, 25$ **f** $2, 5, 8, \ldots, 29$

g $8, 13, 18, \ldots, 53$ **h** $1, 5, 9, \ldots, 37$

3 A and vi, B and iv, C and i, D and v, E and ii, F and iii

4 B is greater. [Value of A is 92, value of B is 93]

5 $\frac{1}{4} \times 1 + 8 = 8\frac{1}{4}, \frac{1}{4} \times 2 + 8 = 8\frac{1}{2}, \frac{1}{4} \times 3 + 8 = 8\frac{3}{4}, \frac{1}{4} \times 4 + 8 = 9$

6 a $6\frac{1}{2}, 7, 7\frac{1}{2}, \ldots, 10$ **b** $2\frac{1}{2}, 7\frac{1}{2}, 12\frac{1}{2}, \ldots, 37\frac{1}{2}$

c $1.7, 1.9, 2.1, \ldots, 3.1$

d $4.25, 8.75, 13.25, \ldots, 35.75$

7 a i $16, 20, 24, 28$ **ii** $1\frac{1}{4}, 1\frac{1}{2}, 1\frac{3}{4}, 2$

    iii $8, 4, 0, -4$ **iv** $\frac{3}{4}, \frac{1}{2}, \frac{1}{4}, 0$

b learners' answers – examples given:

What is similar about the sequences in **ai** and **aii** – terms are increasing

What is similar about the sequences in **aiii** and **aiv** – terms are decreasing

What is different about the sequences in **ai** and **aiii** – in one the terms are increasing, in the other they are decreasing

What is different about the sequences in **aii** and **aiv** – in one the terms are increasing, in the other they are decreasing

c learners' answers – example given:

Group A – increasing terms, $3n + 7, \frac{1}{2}n + 12, \frac{1}{4}n - 19$

Group B – decreasing terms, $13 - n$, $9 - 5n, 15 - \frac{2}{3}n, \frac{7}{8} - \frac{1}{8}n$

8 The sequence $24, 18, 12, 6, 0, \ldots$ is decreasing, but the expression $2n + 22$ is for an increasing sequence.

9 a No, the 14th term is 47 and the 15th term is 50, so 48 is not in the sequence.

b, c, d learners' answers

e i Yes, 39 is the 21st term in the sequence.

    ii No, the 8th term is 55 and the 9th term is 61, so 60 is not in the sequence $6n + 7$.

10 nth term is $2n + 6$

position number (n)	1	2	3	4
term	8	10	12	14
$2 \times n$	2	4	6	8
$2 \times n + 6$	8	10	12	14

11 a $2n + 4$ **b** $3n + 2$

c $5n + 1$ **d** $4n - 1$

e $8n - 6$ **f** $7n - 5$

12 a $1, 4, 7, 10, \ldots$ **b** $3n - 2$

c 58

13 They are both wrong, the expression for the nth term is $\frac{1}{2}n + 3\frac{1}{2}$

14 a $\frac{1}{4}n + 9$ **b** $0.6n + 4$

c $\frac{1}{2}n - 2$ **d** $1.4n - 2$

15 a Arun is correct as the sequence is decreasing.

b nth term is $-2n + 10$, or $10 - 2n$

c peer discussion

16 a $-3n + 21$ or $21 - 3n$

b $-4n + 15$ or $15 - 4n$

c $-5n + 12$ or $12 - 5n$

Exercise 9.4

1 a i

x	1	$2\frac{1}{2}$	4	$5\frac{1}{2}$
y	4	$5\frac{1}{2}$	7	$8\frac{1}{2}$

    ii

x	7	$7\frac{1}{2}$	8	$8\frac{1}{2}$
y	4	$4\frac{1}{2}$	5	$5\frac{1}{2}$

b i

ii

| x | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |

| y | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
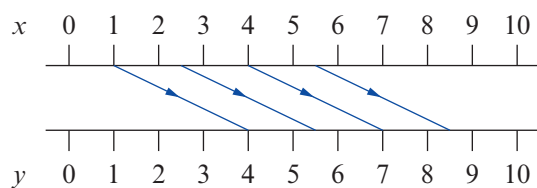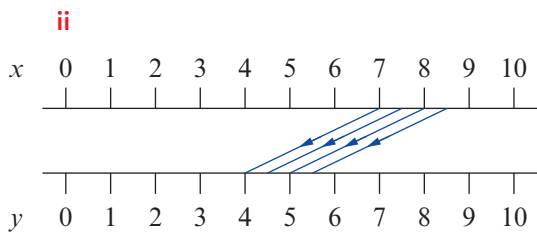
c i $y = x + 3$ **ii** $y = x - 3$

2 a i

x	0	$1\frac{1}{2}$	3	$4\frac{1}{2}$
y	3	6	9	12

ii

x	8	10	15	19
y	1	2	$4\frac{1}{2}$	$6\frac{1}{2}$

b i $y = 2x + 3$ **ii** $y = \frac{x}{2} - 3$

3 a i

x	3	4.5	6	7.5
y	6	9	12	15

ii

x	2	6	13	19
y	3	5	$8\frac{1}{2}$	$11\frac{1}{2}$

b peer discussion

4 a i

x	3	$5\frac{1}{2}$	9	$12\frac{1}{2}$
y	8	$15\frac{1}{2}$	26	$36\frac{1}{2}$

ii

x	4	7	10	13
y	7	$8\frac{1}{2}$	10	$11\frac{1}{2}$

b i $y = 3x - 1$ **ii** $y = \frac{x}{2} + 5$

5 a peer discussion

b

x	10	19	28
y	2	5	8

6 a

$x \rightarrow \boxed{+2} \rightarrow y$ equation: $y = x + 2$

$x \leftarrow \boxed{-2} \leftarrow y$ reverse equation: $x = y - 2$

b

$x \rightarrow \boxed{\div 4} \rightarrow y$ equation: $y = \frac{x}{4}$

$x \leftarrow \boxed{\times 4} \leftarrow y$ reverse equation: $x = 4y$

c

$x \rightarrow \boxed{-3} \boxed{\times 8} \rightarrow y$ equation: $y = 8(x - 3)$

$x \leftarrow \boxed{+3} \boxed{\div 8} \leftarrow y$ reverse equation: $x = \frac{y + 3}{8}$

d

$x \rightarrow \boxed{\div 4} \rightarrow y$

$x \leftarrow \boxed{\times 4} \leftarrow y$

e

$x \rightarrow \boxed{-3} \boxed{\times 8} \rightarrow y$

$x \leftarrow \boxed{+3} \boxed{\div 8} \leftarrow y$

f

$x \rightarrow \boxed{\div 2} \boxed{-7} \rightarrow y$

$x \leftarrow \boxed{\times 2} \boxed{+7} \leftarrow y$
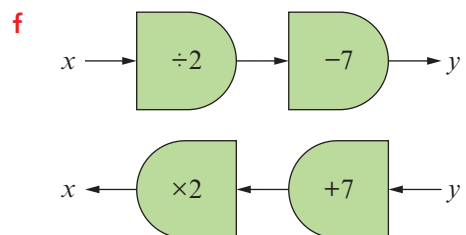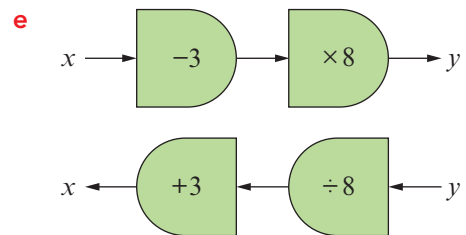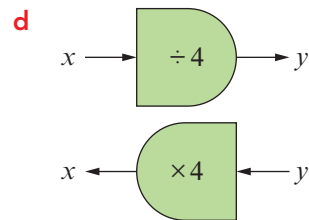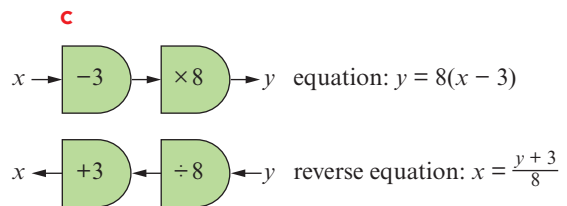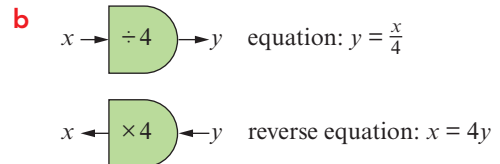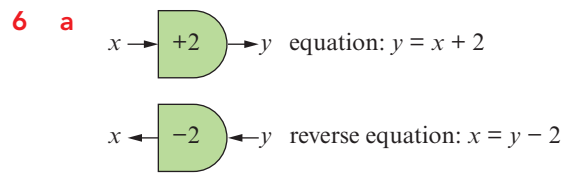
7 A and v, B and i, C and iv, D and vi, E and iii, F and ii

8 a i

$x \rightarrow \boxed{+9} \rightarrow y$

ii

$x \rightarrow \boxed{\times 3} \rightarrow y$
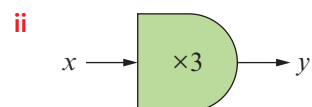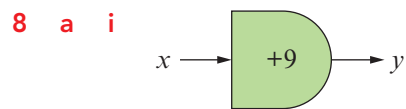
b i $y = x + 9$ **ii** $y = 3x$

9 Arun is correct

10 $y = 3x + 2$

11 $y = \frac{1}{2}x + 9$

Check your progress

1 a i add $\frac{1}{3}$ **ii** $3\frac{1}{3}, 3\frac{2}{3}$

  b i subtract 0.3 **ii** 5.5, 5.2

2 a 3, 5, 7, . . . **b** add 2

  c
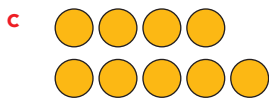
  d

position number	1	2	3	4
term	3	5	7	9
2 × position number	2	4	6	8
2 × position number + 1	3	5	7	9

Position-to-term rule is:
term = 2 × position number + 1

3 a $9, 9\frac{1}{2}, 10, \ldots, 13\frac{1}{2}$

  b 4.25, 9.25, 14.25, . . . , 49.25

4 a $2n + 7$ **b** $18 - 3n$

5 a i

x	8	10	20	30
y	1	$1\frac{1}{2}$	4	$6\frac{1}{2}$

   ii

x	-5	$-\frac{1}{2}$	2	$5\frac{1}{2}$
y	8	17	22	29

  b i $y = \frac{x}{4} - 1$ **ii** $y = 2(x + 9)$

Unit 10

Getting started

1 a 62% **b** 38%

2 a 240 **b** 840 **c** 3

3 25% is $\frac{1}{4}$; 0.25% is $\frac{1}{4}$ of 1% or $\frac{1}{400}$

4 a 10% **b** 8%

5

100%	10%	60%	120%	350%
$850	$85	$510	$1020	$2975
4.50 m	0.45 m	2.70 m	5.40 m	15.75 m

Exercise 10.1

1 a $10.50 **b** $80.50 **c** $59.50

2 a 2560 people **b** 5760 **c** 640

3 a 0.38 kg **b** 19.38 kg

  c 18.62 kg

4 a $264 **b** $360 **c** $408

  d $480 **e** $528

5 83 250

6 a $1.35 \times 60 = 81$ **b** $1.9 \times 32 = 60.8$

  c $2.2 \times 140 = 308$

7 a 40% **b** 250%

8 a 20% **b** 500%

9 a 0.009 m **b** 1.809 m

10 a 12 km **b** 54 km

   c 102 km **d** 156 km

11

Amount	40%	140%	280%	420%
$20	$8	$28	$56	$84
50 kg	20 kg	70 kg	140 kg	210 kg
90 m	36 m	126 m	252 m	378 m

12 a 33 kg **b** 55 kg

13 a $196, $364, $728 **b** $552

14 A $448.20   B $679.32   C $1501.20

15 Ace $15 484   Beta $18 744   Carro $24 079

16 a Mia correctly found that 70% of $120 is $84 but she did not subtract this from $120

   b $36

17 a 1500; it increases to 3000 and then it halves to 1500

   b It increases to 2500 and then decreases to 1875

   c learners' answers

   d peer discussion

18 a $88

b Arun subtracted $8 which is incorrect, Sofia found 10% of $88 which is $8.80 and then found $88 − $8.80 = $79.20

c $79.20

19 a $480

b 20% of $480 = $96 so the price now is $480 + $96 = $576; it is more than $560

c peer discussion

20 a A decrease of $750 gives − $250 left. This could mean she is $250 in debt, so it is possible.

b If he only had 500 g he cannot have 750 g less. The statement is false.

Exercise 10.2

1 a 1.63   **b** 0.37

c 2.03   **d** 0.12

2 A and ii; B and iii; C and i; D and vi; E and v; F and iv

3 a 1.45   **b** 3.45   **c** 0.55

4 a 25% decrease   **b** 22% increase

c 233% increase   **d** 67% decrease

e 97% decrease

5 a 74   **b** 333   **c** 22.2

6 a 54 kg   **b** 121.5 kg   **c** 18 kg

7 a 42.9 kg   **b** 106.9 kg

8 a 1590.6   **b** 1053.15   **c** 89.6

9 a 252   **b** 268.8

c 198   **d** 297.6

10 a 167%   **b** 62.5%

11 a to d all are 120

12 a 73 944   **b** increase by 74.4%

c 3440

13 a i 49.5%   **ii** 55.9%

iii 78.3%   **iv** 147.1%

b 1960 to 1970

c Here is one method: If the percentage changes from 2000 to 2010 and from 2010 to 2020 are the same, then the population will be 1452 million.

14

Original price	Percentage reduction	Reduced price
$280	20%	$224
$420	45%	$231
$620	65%	$217
$750	6%	$705

15 a 4.20 m   **b** 8.47 m

c 13.14 m

16 a 11.3%   **b** 29.8%   **c** 57.2%

17 a 407% is 100% + an increase of 307%

b peer discussion

18 a You could find the increase and add it or you could use a multiplier.

b learners' own answers

Check your progress

1 a 40%   **b** 250%

2 a increase = $0.35 \times 240 = 84$ so the value is $240 + $84 = $324

b $240 \times 1.35 = $324

3 a 175% increase   **b** 665.5 million

Unit 11

Getting started

1 a $85   **b** $y = x - 25$

2 $y = 80d$

3 a

x	−3	−2	−1	0	1	2	3
y	−1	0	1	2	3	4	5

b graph plotted from the table of values

4 a The line goes down from left to right.

b 5 minutes

Exercise 11.1

1 a i $22   **ii** $40

iii $76

b $c = 6b + 4$

2 a i $75 **ii** $135

iii $90

b $c = 30h + 45$

3 a i $51 **ii** $75

iii $123

b $d = 12t + 3$

4 a i $180 **ii** $280

iii $430

b $y = 50w + 80$

5 a $83 **b** $c = 2n + 3$ **c** 23

6 a $45 \times 7 + 25 = 315 + 25 = 340$

b The cost is $45 \times 14 + $25 = 655

c $a = 45n + 25$

7 a i 1.9 m **ii** 2.3 m

b 10 weeks because $1.5 + 10 \times 0.2 = 3.5$

c $h = 0.2t + 1.5$

8 a i 18 **ii** 24

b $z = 0.5f - 2$ **c** 64

9 a i 12 **ii** 3

iii 18 **iv** 0

b r is negative **c** $0 \leqslant t \leqslant 6$

10 a Adding the sides in order from the top left corner, $p = 8 + L + 3 + 6 + 3 + 2 + L = 2L + 22$

b $a = 8L + 18$

11 a i 99 litres **ii** 81 litres

iii 45 litres

b 12 hours

c $l = 108 - 9h$

12 A, B, D and E

13 a $2 \times 4 - 2 = 6$ **b** $0.5 \times 4 + 4 = 6$

c learners' own answers

14 The fixed charge is $40 and the amount for each person is $8.

15 a learners' own answers

b peer discussion

Exercise 11.2

Tables of values are given for graph checking throughout this exercise.

1 a

x	−2	−1	0	1	2	3
y	−1	1	3	5	7	9

b graph plotted from the table of values $y = 2x + 3$

2 a

x	−2	−1	0	1	2	3
y	−4	−1	2	5	8	11

b graph plotted from the table of values $y = 3x + 2$

3 a

x	−3	−2	−1	0	1	2
y	0	2	4	6	8	10

b and **c** graph plotted from the table of values $y = 2x + 6$; graph line extended to pass through (3, 12)

d If $x = 3$ then $y = 2 \times 3 + 6 = 12$

4 a

x	−3	−2	−1	0	1	2	3
y	6	4	2	0	−2	−4	−6

b graph plotted from the table of values $y = -2x$

5 a

x	−2	−1	0	1	2	3	4	5
y	6	5	4	3	2	1	0	−1

b graph plotted from the table of values $y = -4 - x$

c at (4, 0) **d** at (0, 4)

e i $4 - 10 = -6$

ii It is when the line is extended.

6 a

x	−2	−1	0	1	2	3	4
y	10	20	30	40	50	60	70

b graph plotted from the table of values $y = 10x + 30$

c at (0, 30)

d i 55 **ii** yes (2.5, 55)

e (10, 30) is on the line because $10 \times 10 + 30 = 130$ but (20, 260) is not because $10 \times 20 + 30 = 230$

f peer discussion

7 a

x	−10	0	10	20	30	40
y	20	40	60	80	100	120

   b graph plotted from the table of values
$y = 2x + 40$

   c at (0, 40)   **d** all of them

   e peer discussion

8 a i (4, 5)   **ii** (7, 20)

    iii (0, −15)   **iv** (20, 85)

    v (3, 0)

   b at (0, −15)   **c** at (3, 0)

9 a $2 per day for n days is $2n$ and then you add the delivery charge of $5

   b

n	0	1	2	3	4	5	6	7
c	5	7	9	11	13	15	17	19

   c graph plotted from the table of values
$c = 2n + 5$

   d Because the cost and the number of days cannot take negative values.

10 a In h hours the diesel used is $5h$ litres and so there are $40 - 5h$ litres remaining.

   b

h	0	1	2	3	4	5	6
f	40	35	30	25	20	15	10

   c graph plotted from the table of values
$f = 40 - 5h$

   d The line starts at 40 on the y-axis.

   e 8 hours

11 a $y = 15d + 35$

   b

d	0	1	2	3	5	8
y	35	50	65	80	110	155

   c graph plotted from the table of values
$y = 15d + 35$

   d The graph goes through (7, 140) so the cost is $140; using the formula, if $d = 7$ then $y = 15 \times 7 + 35 = 140$ which gives the same answer.

   e The line starts at 35 on the y-axis.

12 a $c = 20n + 60$

   b

n	0	1	2	3	4	5	6
c	60	80	100	120	140	160	180

   c graph plotted from the table of values
$c = 20n + 60$

   d Zara is not correct. The cost in dollars for 12 hours is $20 \times 12 + 60 = 300$ but twice the cost for 6 hours is $2 \times 180 = 360$

13 a $y = 2x + 10$

   b

x	0	1	2	3	4	5	6
y	10	12	14	16	18	20	22

   c line B

   d Plant A: the initial height is 10 cm and it grows 1 cm per week. Plant B: the initial height is 10 cm and it grows 5 cm per week.

   e peer assessment

14 a

x	−3	−2	−1	0	1	2	3
y	−1	1	3	5	7	9	11

   b graph plotted from the table of values
$y = 2x + 5$

   c i at (0, 5)   **ii** at (−2.5, 0)

   d learners' own answers

   e The points are (0, 2) and (−1, 0)

Exercise 11.3

1 a

x	−3	−2	−1	0	1	2	3
$2x$	−6	−4	−2	0	2	4	6
$2x + 4$	−2	0	2	4	6	8	10
$2x - 3$	−9	−7	−5	−3	−1	1	3

   b i, ii and **iii** graphs plotted from table of values, drawn on same axes (parallel lines through intercepts 0, 4 and −3)

   c i gradient = 2, intercept = 0

    ii gradient = 2, intercept = 4

    iii gradient = 2, intercept = −3

   d i learners' examples: any equation of the form $y = 2x + c$ where c is an integer

    ii learners' conjectures, e.g. it should be parallel to the other lines

    iii learners' own graphs

2 **a**

x	−3	−2	−1	0	1	2	3
$y = 3x$	−9	−6	−3	0	3	6	9
$y = 3x + 3$	−6	−3	0	3	6	9	12
$y = 3x − 1$	−10	−7	−4	−1	2	5	8

graphs plotted from table of values, drawn on same axes (parallel lines through intercepts 0, 3 and −1)

   b All gradients = 3, intercepts = 0, 3 and −1

   c any graph of an equation of the form $y = 3x + c$ where c is a number

3 learners' investigation leading to a conclusion that all the gradients = 4, and lines are parallel

4 **a** and **b**

x	−2	−1	0	1	2	3	4	5
$y = −x + 5$	7	6	5	4	3	2	1	0
$y = −x + 2$	4	3	2	1	0	−1	−2	−3

   c graphs plotted from table of values, drawn on same axes

   d −1

   e any equation of the form $y = −x + c$ where c is a number

5 **a** and **b**

x	−2	−1	0	1	2	3	4
$−2x + 9$	13	11	9	7	5	3	1
$−2x + 6$	10	8	6	4	2	0	−2

   c graphs plotted from table of values, drawn on same axes

   d −2

   e any equation of the form $y = −2x + c$ where c is a number

6 **a** First group: $y = 4x + 6$ and $y = 4x + 2$
Second group: $y = 6x + 4$ and $y = 6x + 2$
Third group: $y = 2x + 4$ and $y = 2x + 6$

   b First group, any equation of the form $y = 4x + c$; second group, any equation of the form $y = 6x + c$; third group, any equation of the form $y = 2x + c$

7 **a** $y = 5x + 12$ and $y = 5x − 6$

   b $y = 5x$

8 **a**

x	−2	−1	0	1	2	3
$x + 3$	1	2	3	4	5	6
$2x + 3$	−1	1	3	5	7	9
$−x + 3$	5	4	3	2	1	0

   b and **c** graphs plotted from table of values, drawn on same axes (common intercept = 3)

   d gradients are 1, 2, −1

   e the y-intercept is 3 for all three lines

   f any equation of the form $y = mx + 3$ where m is a number

9 **a**

x	−2	−1	0	1	2	3
$x − 2$	−4	−3	−2	−1	0	1
$3x − 2$	−8	−5	−2	1	4	7
$−2x − 2$	2	0	−2	−4	−6	−8

   b graphs plotted from table of values, drawn on same axes (common intercept −2)

   c gradients are 1, 3, −2

   d y-intercept is −2 for all three lines

   e any equations of the form $y = mx − 2$ where m is a number

10 **a**

x	0	1	2	3	4
y	40	65	90	115	140

   b graph plotted from table of values

   c the y-intercept is 40 and the gradient is 25

   d the y-intercept is the fixed charge and the gradient is the charge per hour

11 **a**

x	0	1	2	3	4	5	6	7
y	200	300	400	500	600	700	800	900

   b graph plotted from table of values

   c the cost of the flights is the y-intercept

   d the gradient shows the charge for each night in the hotel

12 a

x	0	1	2	3	4	5	6	7	8
y	40	37	34	31	28	25	22	19	16

    b   graph plotted from table of values

    c   the number of litres at the start

    d   the number of litres flowing out every minute

13 a

x	0	1	2	3	4	5	6
y	30	40	50	60	70	80	90

    b   graph plotted from table of values

    c   learners' own description of a situation with a fixed value/price of 30 and an increase of 10 per unit; 10 is the gradient and 30 is the y-intercept

    d   peer assessment

14 a & d
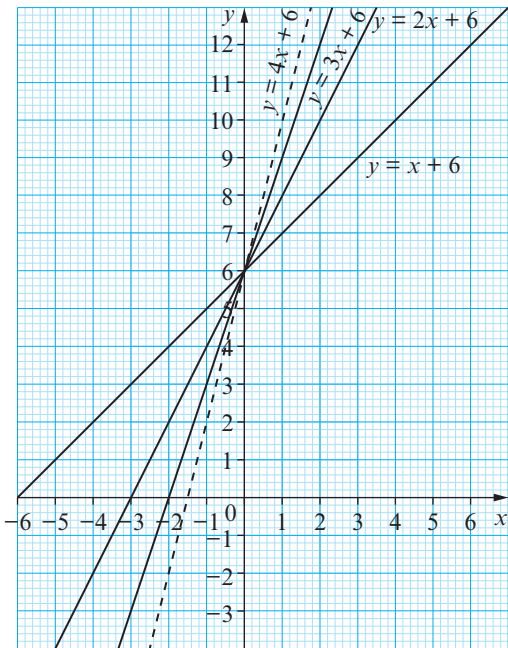
    b   6 for each line   **c** $-6, -3$ and -2

    d -1.5 **e** $-\dfrac{6}{m}$

Reflection

Here are two possible conclusions: lines with the same value of m are parallel; lines with the same value of c have the same y-intercept

Exercise 11.4

1 **a** 09:30   **b** 20 km   **c** 1 hour

2 **a** 1.5 hours   **b** 1.5 hours

    c 3 hours   **d** about 130 km

3 **a** the line of Jake is steeper

    b 2 minutes

    c at the start of a lap

4 **a** **i** 10 m/s   **ii** 60 seconds

    b 25 m/s

    c
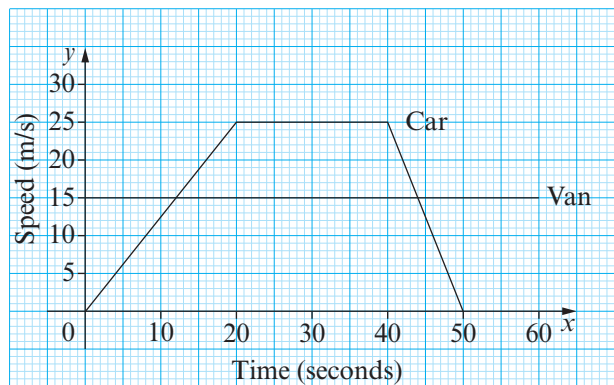
    d peer assesment

    e after 12 seconds and 44 seconds

    f 32 seconds

5 **a** B   **b** 3 hours

    c

hours	0	1	2	3	4	5	6
cost ($)	200	230	260	290	320	350	380

    d $200   **e** $30

    f $100   **g** $50

6 **a** tariff A   **b** 3 minutes

    c 4 minutes

    d 150 rupees for A and 50 rupees for B

    e 25 rupees for A and 50 rupees for B

7 **a** Y because the line is steeper

    b 20 cm

    c after 2 weeks

    d 4 cm

8 **a** car 15 litres and van 20 litres

    b car 7 litres and van 14 litres

c the vehicles both have 10 litres of fuel when they have travelled 50 kilometres

9 a Arun 10 km, Marcus 50 km

b 25 km

c $y = 5x + 10$

d He is 50 km from home at the start. He walks for 3 hours at 6 km/h. He stops after 3 hours and stays where he is for 2 hours.

e They meet after about 4.4 hours when they are 32 km from home.

10 a Liquid A is initially at 40 °C. It cools at a rate of 2.5 degrees per minute until the temperature is 20 °C after 8 minutes. Liquid B is initially at 10 °C. It heats up at a rate of 10 degrees per minute for 3 minutes until the temperature is 40 °C. Then it cools down at a rate of 5 degrees per minute for 5 minutes until the temperature is 15 °C.

b after about 2.4 minutes and 6 minutes

Check your progress

1 a $1250

b $c = 150n + 200$

2 a $p = 2w + 10$

b

w	0	1	2	3	4	5	6
p	10	12	14	16	18	20	22

c graph plotted from table of values

3 a

x	−3	−2	−1	0	1	2	3
y	−3	0	3	6	9	12	15

b graph plotted from table of values
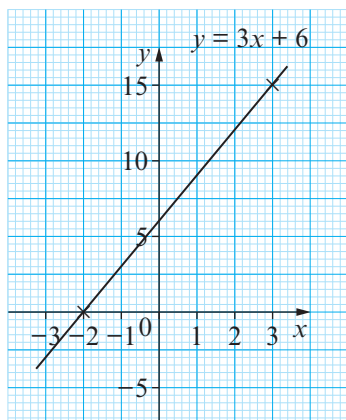
c gradient is 3 and y-intercept is 6

d $y = 3x$

4 a At (0, 12)   **b** −2

5 a Initially, the depth is 10 cm. It increases for 3 minutes at a rate of 5 cm per minute until the depth is 25 cm. The depth does not change after that.

b after 1.8 minutes

c $d = 30 - 6t$

Unit 12

Getting started

1 a 1 : 3   **b** 2 : 3

2 a 1 : 2   **b** 3 : 1

c 2 : 3   **d** 4 : 3

3 a $6 : $12   **b** $10 : $15

4 a 4 m = 400 cm   **b** 6.5 cm = 65 mm

c 5 t = 5000 kg   **d** 0.8 kg = 800 g

e 2.3 l = 2300 ml   **f** 0.75 km = 750 m

Exercise 12.1

1 a 1 : 5   **b** 1 : 6   **c** 1 : 5

d 6 : 1   **e** 3 : 1   **f** 9 : 1

g 2 : 3   **h** 3 : 5   **i** 2 : 7

j 15 : 2   **k** 18 : 5   **l** 5 : 4

2 a 1 : 2 : 3   **b** 4 : 5 : 6   **c** 4 : 3 : 5

d 6 : 5 : 1   **e** 3 : 1 : 5   **f** 9 : 2 : 4

3 a He has only compared and simplified the first two numbers in the ratio, not all three.

b 2 : 4 : 1

4 a learners' answers

Ideally Sofia's method as it's better to change the cm to mm first so both the numbers in the ratio are whole numbers rather than decimals.

b peer discussion

5 a 1 : 2   **b** 3 : 5   **c** 1 : 3

d 2 : 1   **e** 5 : 1   **f** 8 : 3

g 5 : 2   **h** 2 : 3

6 Zara is wrong. The ratio of oranges to sugar is 750 : 1500 = 1 : 2

7 **a** 30 : 50 : 1   **b** 3 : 4 : 6

   c 1 : 7 : 3   **d** 4 : 2 : 1

   e 6 : 5 : 50   **f** 5 : 1 : 4

8 They are both wrong. The ratio of white to red to yellow is 250 : 750 : 1200 = 5 : 15 : 24

9 **a** One bag of sugar. [One bag of sugar = 325 g and one bag of flour = 320 g]

   b A pencil is longer. [One pen = 15 cm and one pencil = 18 cm]

10 learners' answers

   Examples given.

   a To eliminate the decimals from the ratios.

   b It's easier to simplify ratios when they are whole numbers rather than decimals.

   c Start by multiplying both numbers by 100.

   d peer discussion

11 **a** 1 : 4   **b** 1 : 2   **c** 1 : 2

   d 6 : 1   **e** 5 : 1   **f** 3 : 5

   g 2 : 7   **h** 3 : 1 : 2

12 learners' answers

   Examples given.

   a The time for Tuesday is less than the time for Monday but the ratio is more.

   b 1 hour 40 mins is not 1.4 hours and 50 mins is not 0.5 hours.

   c Change all to minutes, so 100 : 50 : 150 = 2 : 1 : 3

Exercise 12.2

1 Total number of parts: 3 + 2 + 5 = 10

   Value of one part: $80 ÷ 10 = 8

   So gets: 3 × 8 = $24

   Luana gets: 2 × 8 = $16

   Kyra gets: 5 × 8 = $40

2 **a** $15 : $30 : $45   **b** $50 : $75 : $100

   c $144 : $240 : $48   **d** $144 : $72 : $180

3 learners' answers

   Example given.

   a Add the individual amounts to check the total is equal to the original amount.

   b peer discussion

4 **a, b** Amounts are for D : E : J

   i $42 : $56 : $70   Check: 42 + 56 + 70 = 168

   ii $48 : $64 : $80   Check: 48 + 64 + 80 = 192

   iii $58.50 : $78 : $97.50   Check: 58.50 + 78 + 97.50 = 234

5 **a** **i** 95   **ii** 133   **iii** 57

   b 38   **c** 38

6 **a** **i** 32   **ii** 16   **iii** 24

   b **i** 27   **ii** 9   **iii** 36

7 Aden $150, Eli $100, Lily $75, Ziva $125

8 Share $150 in the ratio 2 : 3 : 1 = $50 : $75 : $25

   Share $120 in the ratio 3 : 1 : 4 = $45 : $15 : $60

   Share $132 in the ratio 1 : 5 : 6 = $11 : $55 : $66

   Share $126 in the ratio 2 : 6 : 1 = $28 : $84 : $14

9 36° : 54° : 90°

10 learners' answers

   Examples given.

   a Simplify the ratio to 2 : 3 : 5

   b 2 + 3 + 5 = 10, 2520 ÷ 10 = $252

   252 × 2 = $504, 252 × 3 = $756, 252 × 5 = $1260

   c and d peer discussion

11 This year: 4 + 9 + 11 = 24, 300 ÷ 24 = 12.5, oldest child gets 11 × 12.5 = $137.50

   In two years' time: 6 + 11 + 13 = 30, 300 ÷ 30 = 10, oldest child gets 13 × 10 = $130

   $137.50 − $130 = $7.50

12 Lin pays $30 000, so ratio Zhi : Zhen : Lin = 2 : 3 : 1

$228 000 ÷ 6 = 38 000, Lin gets $38 000 from sale, so profit is $8000

Lin is incorrect.

13 a learners' answers

b Height is better for Bishara.

Using age, Bishara gets $300. Using height, Bishara gets $304.

Exercise 12.3

1 a Total number of parts $= 2 + 7 = 9$

   i fraction that are cashew nuts $= \frac{2}{9}$

   ii fraction that are peanuts $= \frac{7}{9}$

b Total number of parts $= 3 + 5 = 8$

   i fraction that are plastic $= \frac{3}{8}$

   ii fraction that are paper $= \frac{5}{8}$

c Total number of parts $= 3 + 1 = 4$

   i fraction that are apples $= \frac{3}{4}$

   ii fraction that are bananas $= \frac{1}{4}$

2 a peer discussion

b peer discussion

3 a $\frac{3}{10}$ **b** 15

4 a $\frac{3}{7}$ **b** 15

5 learners' answers

Example given.

Zara is correct but Sofia isn't. As the ratio is 5 : 4, the number of counters must be a multiple of the total number of parts. $5 + 4 = 9$

62 is not a multiple of 9. 72 is a multiple of 9.

6 D 36. This is the only option that is a multiple of 12.

7 24

8 a 2 : 1   **b** 4 : 3   **c** 5 : 4

9 a Sky blue $\frac{2}{5}$, Sea blue $\frac{3}{10}$

b Sky blue is lighter as it contains a greater proportion of white paint: $\frac{4}{10} > \frac{3}{10}$

10 a Angelica $\frac{5}{8}$, Sanjay $\frac{11}{16}$

b Sanjay, because $\frac{11}{16} > \frac{10}{16}$

11 a $\frac{17}{30}$ **b** $\frac{23}{40}$

c Sharks. Seals $\frac{17}{30} = \frac{68}{120}$, Sharks $\frac{23}{40} = \frac{69}{120}$ and $\frac{69}{120} > \frac{68}{120}$

12 a peer discussion [Lin has the greater proportion of black counters]

b peer discussion

13 Hannah has the greater proportion of stamps.

Liam has 20 coins and 320 stamps, this is the same as 15 coins and 240 stamps.

Hannah has 15 coins and 270 stamps.

14 Mega-Jewellery has the greater proportion of watches.

Bright Jewellery has 12 watches and 180 rings, which is the same as 30 watches and 450 rings

Mega-Jewellery has 30 watches and 438 rings.

Check your progress

1 a 12 : 1   **b** 1 : 5   **c** 1 : 4

2 One bag of peanuts. [One bag of peanuts = 275 g and one bag of walnuts = 270 g]

3 a 104   **b** 65   **c** 91

4 a $\frac{4}{9}$ **b** 20

5 a Silver grey $\frac{5}{7}$, Stone grey $\frac{8}{11}$

b Stone grey as it has the greater fraction of white. Silver grey $\frac{5}{7} = \frac{55}{77}$, Stone grey $\frac{8}{11} = \frac{56}{77}$

Unit 13

Getting started

1 a $\frac{1}{5}$ **b** $\frac{2}{5}$

2 a $\frac{1}{6}$ **b** $\frac{1}{6}$ **c** $\frac{1}{2}$ **d** $\frac{2}{3}$

3 35%

4 $\frac{87}{222} = 0.392$ to 3 d.p.

Exercise 13.1

1 a 0.7   **b** 0.9   **c** 0.6   **d** 0.4

2 a 45%   **b** 75%   **c** 80%

3 a 0.68   **b** 0.83

4 a There are some toys that are other colours.

b i 75% **ii** 90%

iii 70% **iv** 85%

5 a $\frac{1}{36}$ **b** $\frac{10}{36} = \frac{5}{18}$ **c** $\frac{25}{36}$

6 a

Second dice						
6	7	8	9	10	11	12
5	6	7	8	9	10	11
4	5	6	7	8	9	10
3	4	5	6	7	8	9
2	3	4	5	6	7	8
1	2	3	4	5	6	7
	1	2	3	4	5	6

First dice

b i $\frac{2}{36} = \frac{1}{18}$ **ii** $\frac{6}{36} = \frac{1}{6}$

iii $\frac{1}{36}$ **iv** $\frac{4}{36} = \frac{1}{9}$

c

Total	2	3	4	5
Probability	$\frac{1}{36}$	$\frac{1}{18}$	$\frac{1}{12}$	$\frac{1}{9}$
Total	6	7	8	9
Probability	$\frac{5}{36}$	$\frac{1}{6}$	$\frac{5}{36}$	$\frac{1}{9}$
Total	10	11	12	
Probability	$\frac{1}{12}$	$\frac{1}{18}$	$\frac{1}{36}$	

7 a i $\frac{10}{36} = \frac{5}{18}$ **ii** $\frac{26}{36} = \frac{13}{18}$ **iii** $\frac{6}{36} = \frac{1}{6}$

iv $\frac{30}{36} = \frac{5}{6}$ **v** $\frac{15}{36} = \frac{5}{12}$

b One possible answer is getting a 4 or a 5; another is getting a 7 or a 12; there are many others.

c peer assessment

8 a

Dice

Coin		1	2	3	4	5	6
	H	H1	H2	H3	H4	H5	H6
	T	T1	T2	T3	T4	T5	T6

b 12 outcomes; yes, they are all equally likely

c i $\frac{1}{12}$ **ii** $\frac{1}{12}$

iii $\frac{3}{12} = \frac{1}{4}$ **iv** $\frac{2}{12} = \frac{1}{6}$

d i $\frac{11}{12}$ **ii** $\frac{11}{12}$

iii $\frac{3}{4}$ **iv** $\frac{5}{6}$

e There are many possible answers. One is a head but not a 6.

f peer assessment

9 a

Second spinner				
3	×	×	×	×
2	×	×	×	×
1	×	×	×	×
	1	2	3	4
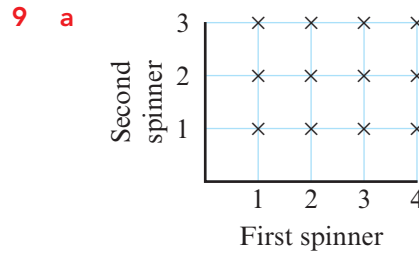
First spinner

Axes can be the other way round.

b i $\frac{1}{12}$ **ii** $\frac{1}{2}$

iii $\frac{3}{12} = \frac{1}{4}$ **iv** $\frac{9}{12} = \frac{3}{4}$

c

Second spinner				
3	4	5	6	7
2	3	4	5	6
1	2	3	4	5
	1	2	3	4

First spinner

d i $\frac{3}{12} = \frac{1}{4}$ **ii** $\frac{3}{12} = \frac{1}{4}$ **iii** $\frac{11}{12}$

iv $\frac{4}{12} = \frac{1}{3}$ **v** $\frac{8}{12} = \frac{2}{3}$

e

Second spinner				
3	3	6	9	12
2	2	4	6	8
1	1	2	3	4
	1	2	3	4

First spinner

f

product	1	2	3	4	6	8	9	12
probability	$\frac{1}{12}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{12}$	$\frac{1}{12}$	$\frac{1}{12}$

g i $\frac{5}{12}$ **ii** $\frac{7}{12}$

iii $\frac{4}{12} = \frac{1}{3}$ **iv** $\frac{8}{12} = \frac{2}{3}$

10 a

		Second coin	
		H	T
First Coin	H	HH	HT
	T	TH	TT

b There are 4 equally likely outcomes. HT and TH are different.

c **i** $\frac{1}{4}$ **ii** $\frac{1}{4}$ **iii** $\frac{1}{2}$

d

First coin Second coin Outcome
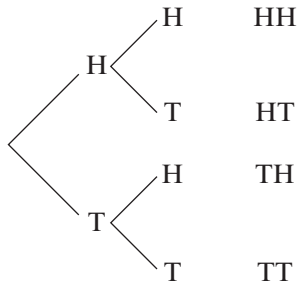
e In the table the outcomes are in the 4 cells. In the tree diagram they are at the ends of the branches.

f There are 8 outcomes: HHH, HHT, HTH, HTT, THH, THT, TTH, TTT

g

First coin Second coin Third coin Outcome
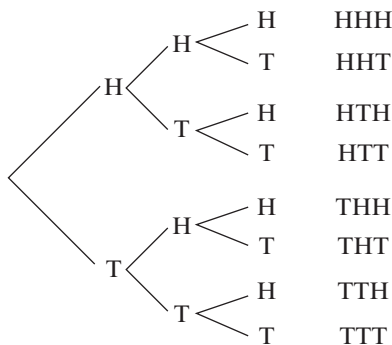
h **i** $\frac{1}{8}$ **ii** $\frac{1}{8}$ **iii** $\frac{7}{8}$

iv $\frac{3}{8}$ **v** $\frac{3}{8}$

11 There are 16 outcomes. One way to find them is to list all the outcomes for 3 coins + H:

HHHH, HHTH, HTHH, HTTH, THHH, THTH, TTHH, TTTH

and then the same again but + T

HHHT, HHTT, HTHT, HTTT, THHT, THTT, TTHT, TTTT

This tree diagram also shows the 16 outcomes, but in a different order.

First Second Third Fourth coin coin coin coin Outcome
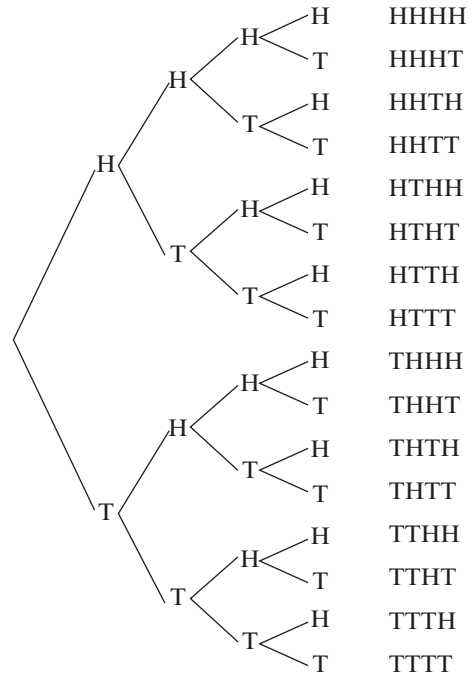
This table shows probabilities:

event	4 heads	3 heads, 1 tail	2 heads, 2 tails
probability	$\frac{1}{16}$	$\frac{4}{16} = \frac{1}{4}$	$\frac{6}{16} = \frac{3}{8}$
event	1 head, 3 tails	4 tails	
probability	$\frac{4}{16} = \frac{1}{4}$	$\frac{1}{16}$	

12 a 245, 254; 425, 452; 524, 542 It helps to list them in order to make sure you have them all.

b **i** $\frac{2}{6} = \frac{1}{3}$ **ii** $\frac{2}{3}$ **iii** $\frac{4}{6} = \frac{2}{3}$

c 24, 25, 28; 42, 45, 48; 52, 54, 58; 82, 84, 85

d **i** $\frac{1}{12}$ **ii** $\frac{11}{12}$ **iii** $\frac{3}{12} = \frac{1}{4}$

iv $\frac{3}{4}$ **v** $\frac{6}{12} = \frac{1}{2}$

e 245, 248, 254, 258, 284, 285; 425, 428, 452, 458, 482, 485; 524, 528, 542, 548, 582, 584; 824, 825, 842, 845, 852, 854

f **i** $\frac{6}{24} = \frac{1}{4}$ **ii** $\frac{3}{4}$ **iii** $\frac{12}{24} = \frac{1}{2}$

Exercise 13.2

1 a 0.4 **b** 0.5

c i 0.567 **ii** 0.45

iii 0.44

d The theoretical probability is 0.5; the experimental values are close to this, sometimes more, sometimes less.

e There are 45 heads so the experimental probability is 0.45; the difference from 0.5 is 0.05, so this is close to the theoretical probability.

2 a 0.5, 0.6, 0.8, 0.7, 0.8

b Based on the first 25 spins and the second 25 spins: 0.56, 0.8

c 0.68 for red, 0.2 for white, 0.12 for blue

d 0.611 for red, 0.273 for white, 0.116 for blue

e He is correct. The values based on 10, 25 or 50 spins vary quite a lot. The values based on 800 are close to the theoretical probabilities of 0.6, 0.3 and 0.1.

3 a Arun 0.7, Sofia 0.45, Marcus 0.775, Zara 0.72

b 0.6917 to 4 d.p.

c 0.666

d The experimental probabilities based on the small samples of the learners vary a lot. The experimental probability based on the learners' combined results is closer to the true value. However, the experimental probability based on 500 throws is very close to the theoretical probability: the difference is 0.0009.

4 learners' own answers

5 learners' own answers

The results should indicate that the likelihood that the experimental probability is close to the theoretical probability increases as the number of throws increases.

Check your progress

1 0.73

2 a 369, 396, 639, 693, 936, 963

b i $\frac{2}{6} = \frac{1}{3}$ **ii** $\frac{4}{6} = \frac{2}{3}$ **iii** 1

3 a

First dice				
4	5	6	7	8
3	4	5	6	7
2	3	4	5	6
1	2	3	4	5
	1	2	3	4
	Second dice			

b i $\frac{3}{16}$ **ii** $\frac{10}{16} = \frac{5}{8}$ **iii** $\frac{4}{16} = \frac{1}{4}$

4 a $\frac{8}{40} = \frac{1}{5}$ or 0.2

b The theoretical probability is $\frac{1}{6}$ or 0.167 to 3 d.p. This is close to 0.2 and you cannot get exactly 0.167 with 40 throws. 7 out of 40 is 0.175 and 6 out of 40 is 0.15.

Unit 14

Getting started

1 a 30°

b correct drawing of an angle of 125°

2 a 9 units **b** 2 units

3 P′ (9, 5)

4

5

6
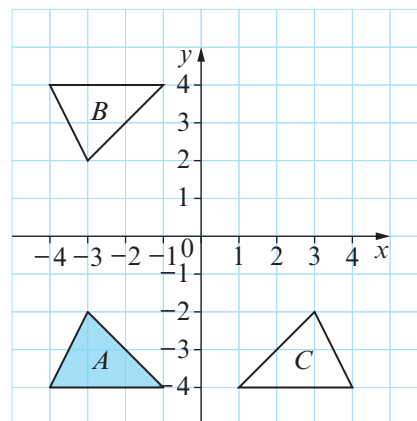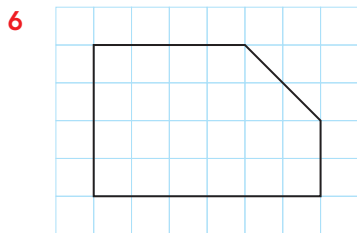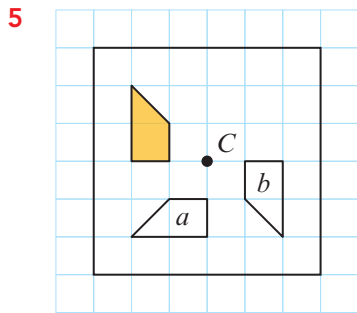
Exercise 14.1

1   **a**   070°   **b**   150°

     c   230°   **d**   300°

2   correct diagrams for these bearings of B from A

     a   025°   **b**   110°

     c   195°   **d**   330°

3   Freya is incorrect. Bearings are always written with three digits so the bearing is 032°, not 32°.

4   **a**   045°   **b**   225°

5   **a**   070°

     b   larger; learners' explanations, e.g. by going north before turning, she has increased the angle between her and the lake

     c   smaller; learners' explanations, e.g. by going south before turning, she has decreased the angle between her and the lake

     d   peer discussion

6   **a**   050°   **b**   165°   **c**   260°

     d   335°   **e**   120°

7   **a**   Bearing of Y from X then X from Y.

       i   058°, 238°   **ii**   142°, 322°

       iii   034°, 214°

     b   learners' diagrams and bearings

     c   learners' answers; connection is 2nd bearing = 1st bearing + 180°.

     d   When the bearing of Y from X is $m°$, the bearing of X from Y is $m + 180°$.

     e   peer discussion

8   **a**   **i**   077°   **ii**   257°

     b   **i**   118°   **ii**   298°

     c   **i**   016°   **ii**   196°

9   **a**   **i**   244°   **ii**   064°

     b   **i**   348°   **ii**   168°

     c   **i**   204°   **ii**   024°

Exercise 14.2

1   *AB* (3, 2), *CD* (3, 4), *EF* (2, 3), *GH* (4, 1)

2   Line segment **AB** and card **iii**, Line segment **CD** and card **iv**, Line segment **EF** and card **vii**, Line segment **GH** and card **i**, Line segment **IJ** and card **vi**, Line segment **KL** and card **viii**, Line segment **MN** and card **ii**, Line segment **PQ** and card **v**.

3   **a**   learners' answers

       Example: advantages – easy to see the midpoint once the diagram is drawn, disadvantages – takes a long time to draw the diagram

     b   learners' answers

       Example: Maha works out the horizontal distance between the points, then halves this distance and adds that value onto the smaller x-coordinate

     c   learners' answers

       Example: advantages – can work it out without having to draw a diagram, disadvantages – as there is no diagram you cannot see if the answer is correct.

     d   learners' answers

     e   learners' answers

       Example: Finding the mean of the x-coordinates (this method is shown in worked example 14.2b, after Q4).

     f   peer discussion

4 a C (7, 4)   b A (7, 2)

   c B$\left(4, 6\frac{1}{2}\right)$ d A$\left(11\frac{1}{2}, 15\right)$

5 a $\left(\frac{2+6}{2}, \frac{3+7}{2}\right) = \left(\frac{8}{2}, \frac{10}{2}\right) = (4, 5)$

   b $\left(\frac{8+12}{2}, \frac{0+6}{2}\right) = \left(\frac{20}{2}, \frac{6}{2}\right) = (10, 3)$

   c $\left(\frac{5+8}{2}, \frac{2+10}{2}\right) = \left(\frac{13}{2}, \frac{12}{2}\right) = \left(6\frac{1}{2}, 6\right)$

   d $\left(\frac{0+7}{2}, \frac{4+11}{2}\right) = \left(\frac{7}{2}, \frac{15}{2}\right) = \left(3\frac{1}{2}, 7\frac{1}{2}\right)$

6 a i (10, 4)   ii $\left(4\frac{1}{2}, 7\frac{1}{2}\right)$

      iii FG$\left(8\frac{1}{2}, 11\frac{1}{2}\right)$

   b learners' diagrams to check answers to part **a**

7 a Hassan has the correct midpoint.

      Shen has added the negative numbers incorrectly in both coordinates.

   b No it doesn't matter in which order you add the *x* and *y* coordinates.

      learners' explanations

      Example: When you are adding two numbers it doesn't matter in which order you add them as you will always get the same answer.

   c peer discussion

8 a $\left(3\frac{1}{2}, -4\right)$ b $\left(-\frac{1}{2}, 2\frac{1}{2}\right)$ c $\left(-8\frac{1}{2}, 7\frac{1}{2}\right)$

9 Midpoint of PR is $\left(\frac{2+2}{2}, \frac{5+-1}{2}\right) = \left(\frac{4}{2}, \frac{4}{2}\right) = (2, 2)$

   Midpoint of QS is $\left(\frac{-2+6}{2}, \frac{3+1}{2}\right) = \left(\frac{4}{2}, \frac{4}{2}\right) = (2, 2)$

   This shows that the diagonals have the same midpoint at (2, 2).

10 Midpoint of DE is (−1, 15), midpoint of DF is $\left(2\frac{1}{2}, -5\right)$, midpoint of EF is $\left(\frac{1}{2}, -10\right)$

11 Let the vertices be A(−2, 1), B(0, 4), C(5, 2) and D(1, −1).

   Diagonal AC has midpoint at $\left(\frac{-2+5}{2}, \frac{1+2}{2}\right) = \left(\frac{3}{2}, \frac{3}{2}\right) = \left(1\frac{1}{2}, 1\frac{1}{2}\right)$

   Diagonal BD has midpoint at $\left(\frac{0+1}{2}, \frac{4+-1}{2}\right) = \left(\frac{1}{2}, \frac{3}{2}\right) = \left(\frac{1}{2}, 1\frac{1}{2}\right)$

   The diagonals do not have the same midpoint.

12 a (6, −3)

   b learners' answers

   c peer discussion

13 (15, −2)

14 CE$\left(\frac{-7+5}{2}, \frac{5+-3}{2}\right) = \left(\frac{-2}{2}, \frac{2}{2}\right) = (-1, 1)$

   AF$\left(\frac{2+-4}{2}, \frac{0+2}{2}\right) = \left(\frac{-2}{2}, \frac{2}{2}\right) = (-1, 1)$

   BD$\left(\frac{-3+1}{2}, \frac{-2+4}{2}\right) = \left(\frac{-2}{2}, \frac{2}{2}\right) = (-1, 1)$

Exercise 14.3

1 A and iii, B and i, C and iv, D and ii.

2

3
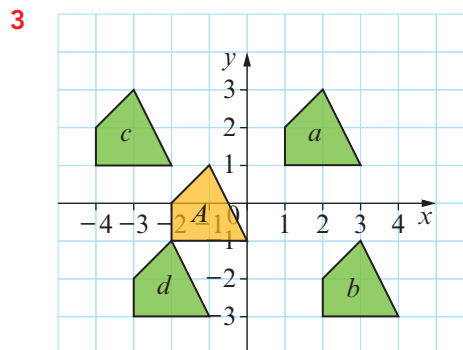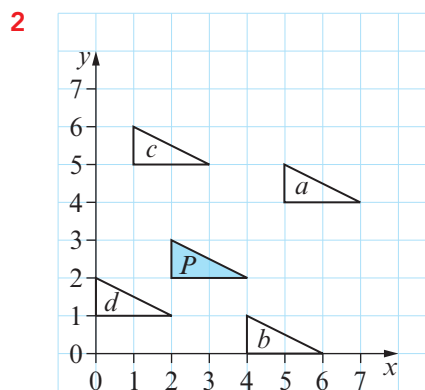
4 a Fin is incorrect.

      learners' explanation

      Example: He needs to reverse $\begin{pmatrix} 3 \\ -2 \end{pmatrix}$ by doing $\begin{pmatrix} -3 \\ 2 \end{pmatrix}$ and not $\begin{pmatrix} 2 \\ -3 \end{pmatrix}$.

   b learners' answers

      Example: Draw a diagram – any triangle will do.

5 a learners' answers

      Example: Draw a diagram – any shape will do.

   b i $\begin{pmatrix} 4 \\ -7 \end{pmatrix}$ ii $\begin{pmatrix} -3 \\ 5 \end{pmatrix}$ iii $\begin{pmatrix} 2 \\ 8 \end{pmatrix}$

c   learners' explanation

Example: Change the signs on the numbers in the vector.

6   a
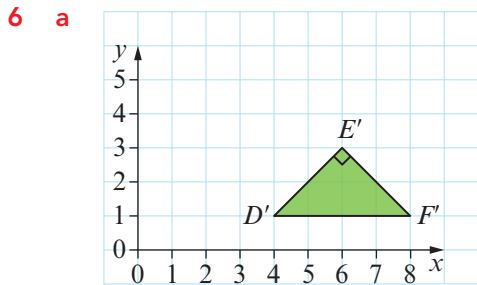
b   ∠D′E′F′ = 90°, ∠D′F′E′ = 45° and ∠E′D′F′ = 45°. D′F′ has a length of 4 units.

c   corresponding lengths are equal

corresponding angles are equal

the object and the image are congruent

7   a   C $\begin{pmatrix} 2 \\ -3 \end{pmatrix}$ b   B $\begin{pmatrix} -2 \\ 3 \end{pmatrix}$

8   a   Translation $\begin{pmatrix} 4 \\ 2 \end{pmatrix}$ b   Translation $\begin{pmatrix} 3 \\ -2 \end{pmatrix}$

c   Translation $\begin{pmatrix} 0 \\ -3 \end{pmatrix}$ d   Translation $\begin{pmatrix} -2 \\ 1 \end{pmatrix}$

e   Translation $\begin{pmatrix} -1 \\ -4 \end{pmatrix}$ f   Translation $\begin{pmatrix} -5 \\ 3 \end{pmatrix}$

9   a   learners' explanation

Example:

Marcus is correct because (2, 0) is how you must write coordinates as they give the position on a grid.

Arun is incorrect because $\begin{pmatrix} 2 \\ 0 \end{pmatrix}$ is a vector and so doesn't give a position on a grid.

b   K′

$(-3, 1) + \begin{pmatrix} 5 \\ -4 \end{pmatrix} = (-3 + 5, 1 + -4)$

$= (2, -3)$

L′

$(-2, 1) + \begin{pmatrix} 5 \\ -4 \end{pmatrix} = (-2 + 5, 1 + -4)$

$= (3, -3)$

c   peer discussion

10   a   A′ (6, 8), B′ (12, 8), C′ (12, 3) and D′ (6, 3)

b   learners' diagrams

c   peer discussion

11   a   i   F′ (0, 6)   ii   G (3, 7)

iii   H (−5, 7)

b   learners' explanations

Exercise 14.4

1   a

b

c

d

e

d

f

e

2 a

f

b
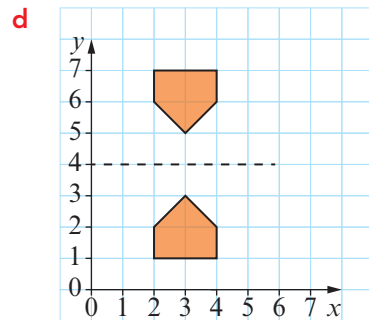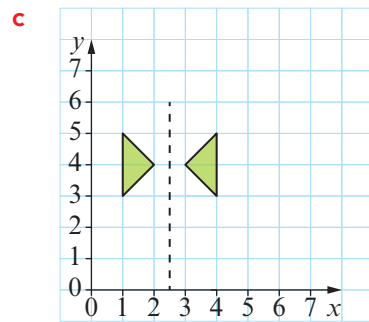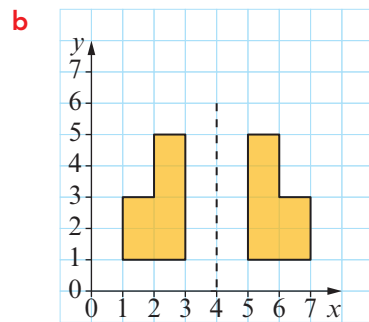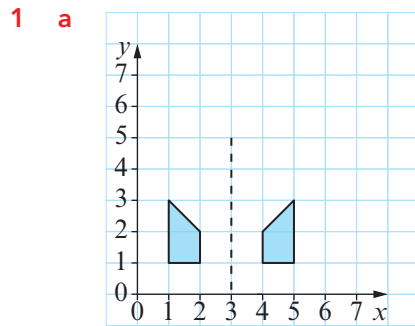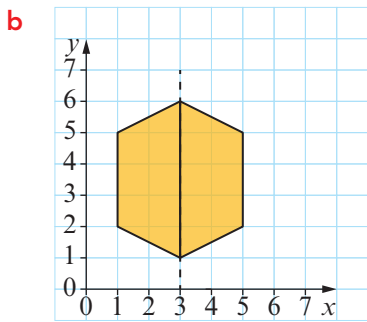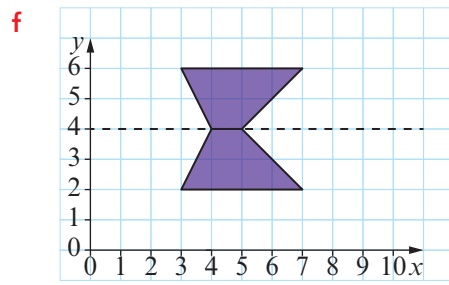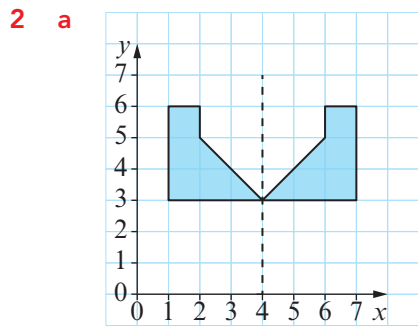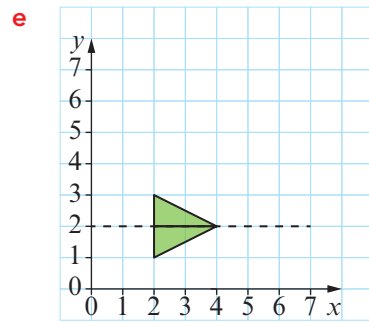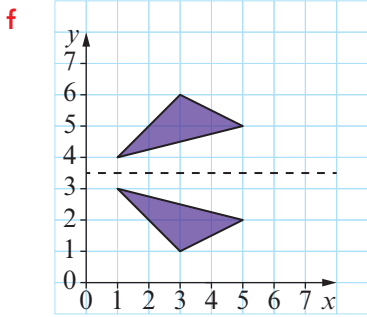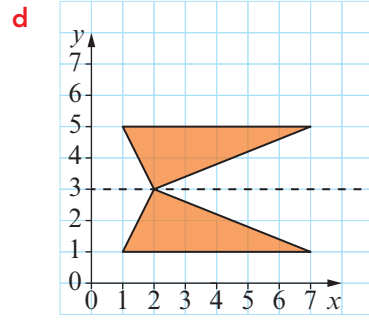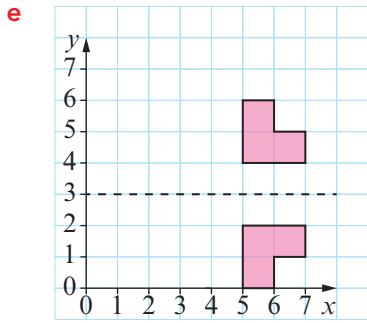
3 a He has reflected the shape in the line $x=-1$ not $y=-1$.

b

c

4

5 a
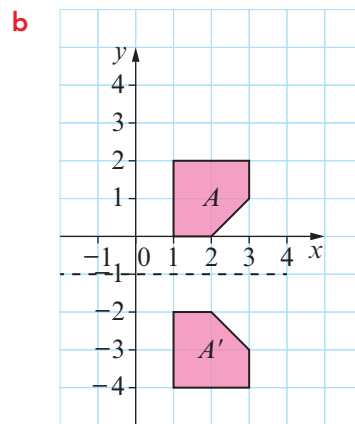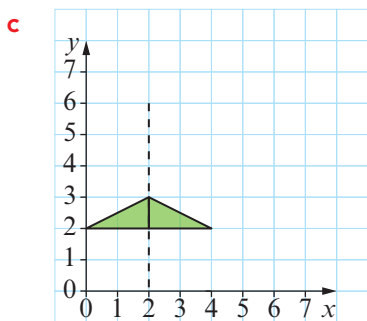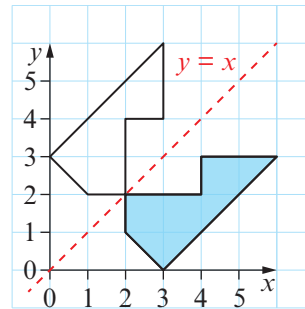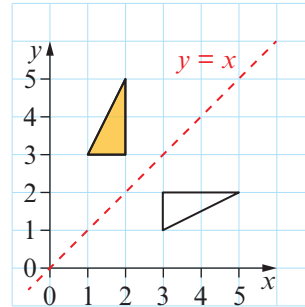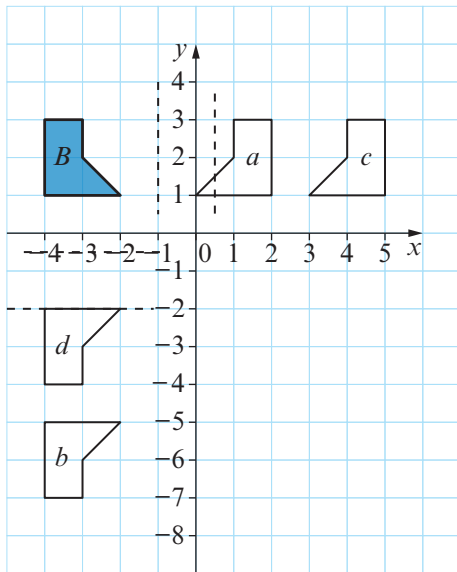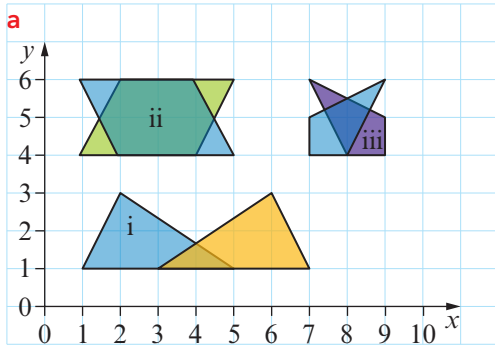
b learners' answers

6 a

b peer discussion

c
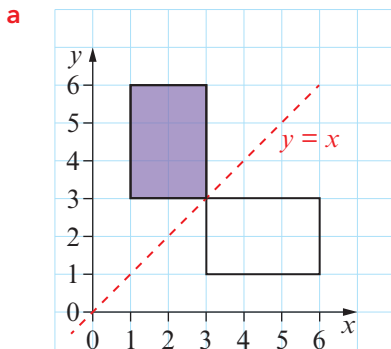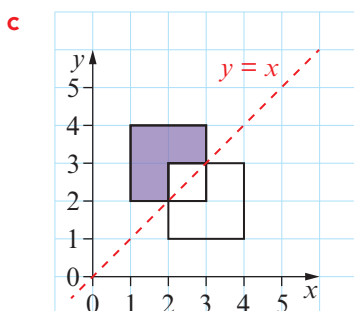
7 a

Object	A (3, 6)	B (3, 4)	C (2, 3)	D (0, 3)
Image	A′ (6, 3)	B′ (4, 3)	C′ (3, 2)	D′ (3, 0)

b learners' answers. Example: The x and y coordinates are reversed.

c learners' answers. Example: Swap the x and y coordinates.

d yes; learners' answers

Example: The line goes diagonally across the grid, and the gradient is positive, so when each point of a shape is reflected across the line, the x coordinate becomes the y coordinate and the y coordinate becomes the x coordinate.

8 a A (2, 5), B (6, 5), C (5, 1) and D (1, 3)

b A′ (5, 2), B′ (5, 6), C′ (1, 5) and D′ (3, 1)

c

d learners' checks

9 a
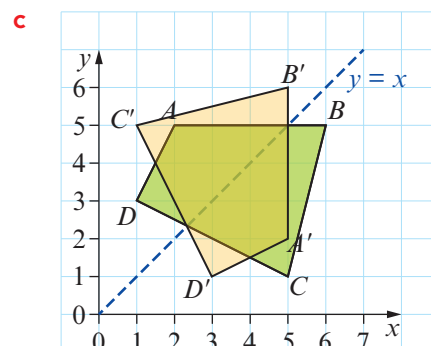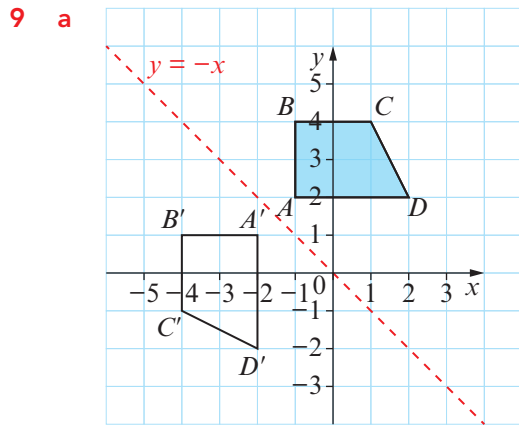
b

Object	A (−1, 2)	B (−1, 4)	C (1, 4)	D (2, 2)
Image	A′ (−2, 1)	B′ (−4, 1)	C′ (−4, −1)	D′ (−2, −2)

c learners' answers

Example: The x and y coordinates are reversed and the signs are changed.

d learners' answers

Example: Swap the x and y coordinates and make the positive numbers negative and the negative numbers positive.

e yes; learners' answers

Example: The line goes diagonally across the grid and the gradient is negative, so when each point of a shape is reflected across the line, the x coordinate becomes the negative of the y coordinate and the y coordinate becomes the negative of the x coordinate.

10 a P (−3, 2), Q (−3, −2) and R (−2, −2)

b P′ (−2, 3), Q′ (2, 3), and R′ (2, 2)

c
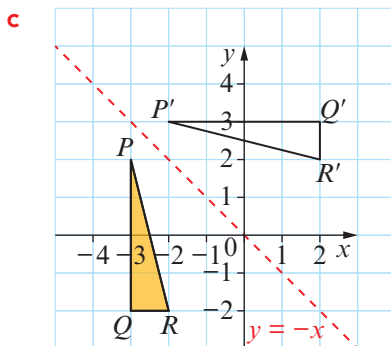
d learners' checks

11 a B $y = 3$ **b** C $x = 5$

c C $y = 3.5$ **d** A $x = 3$

e A $x = 5.5$

12 a reflection in $x = 4$

b reflection in $y = 5$

c not a reflection

d reflection in $y = 4$

e reflection in $x = 5$

f not a reflection

g not a reflection

h reflection in $x = 6$

i reflection in $y = 1$

j not a reflection

Exercise 14.5

1 a

b

c
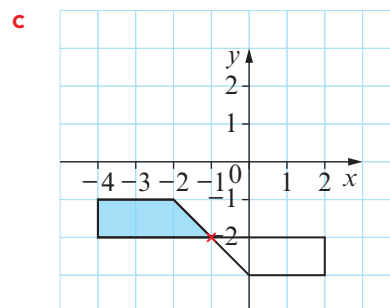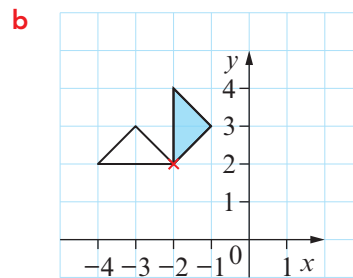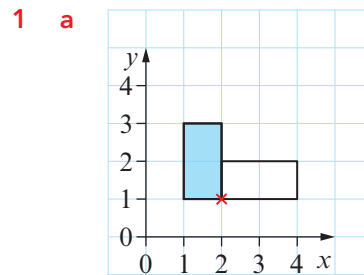
2 a peer discussions

b

3 a i

ii

iii
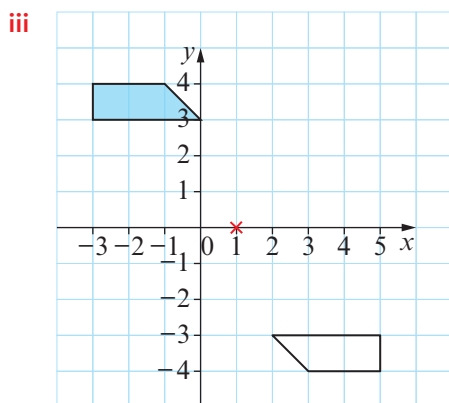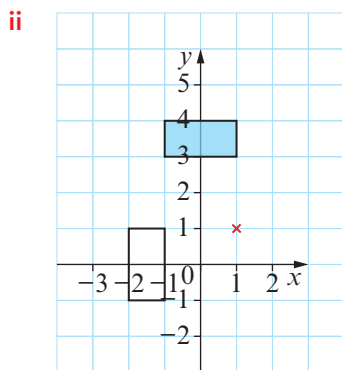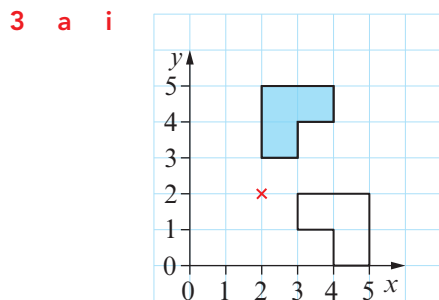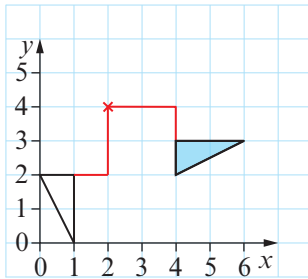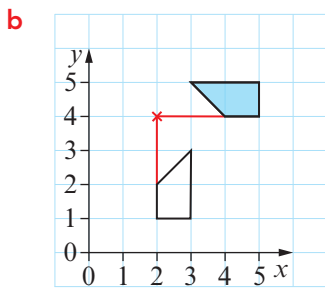
b learners' checks

4 a learners' answers

Example: He has the two points (2, 0) and (2, −2) correct, but then he has drawn the rest of the parallelogram in the wrong direction.

b
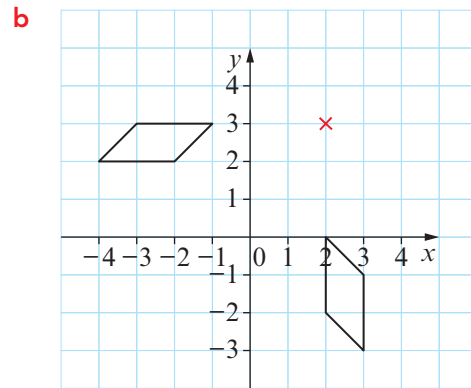
5 a Yes, Sofia and Zara are correct; learners' explanations

Examples: For part **a** Marcus needs to also give the centre of rotation, and for part **b** he doesn't need to say anticlockwise as the direction of the turn doesn't matter for a 180° turn

b learners' answers

Example: Draw lines between corresponding vertices on the object and the image, and the lines cross at the centre of rotation

c i (0, 1)   **ii** (5, 3)

d Example: 'I can find the centre of a 180° rotation by drawing lines between corresponding vertices on the object and the image: the lines cross at the centre of rotation.'

e No it doesn't. These centres are at (3, 2) and (3, −1)

f learners' answers

Example: Use tracing paper and use trial and improvement to find the centre of rotation.

6 a and **iv**, **b** and **i**, **c** and **v**, **d** and **iii**, **e** and **ii**

7 a rotation, 90° anticlockwise, centre (−1, 2)

b rotation, 90° clockwise, centre (−3, 2)

c rotation, 180°, centre (−1, 1)

d rotation, 180°, centre (2, −1)

e rotation, 90° anticlockwise, centre (0, −4)

8 a & b learners' answers

Examples:

Group 1 – (90° clockwise): **i** A to B, **iv** B to D

Group 2 – (90° anticlockwise): **iii** C to B, **vi** D to F

Group 3 – (180°): **ii** A to C, **v** E to B, **vii** G to A

OR

Group 1 – (centre of rotation at (2, 3)): **i** A to B, **ii** A to C, **iii** C to B

Group 2 – (centre of rotation at (5, 2)): **iv** B to D, **v** E to B

Group 3 – (centre of rotation at (6, 5)): **vi** D to F, **vii** G to A

9 a rotation, 90° clockwise, centre (3, 3)

b rotation, 90° clockwise, centre (−2, 2)

c rotation, 90° clockwise, centre (−1, 1)

Exercise 14.6

1 a

b

c

d

e

f
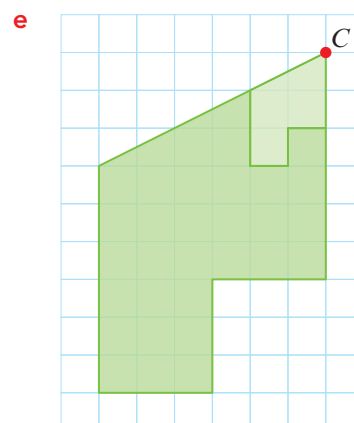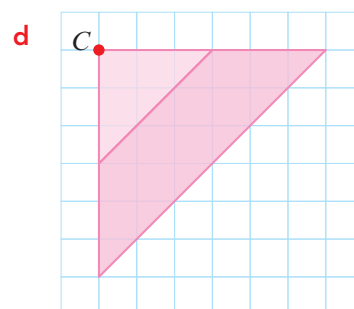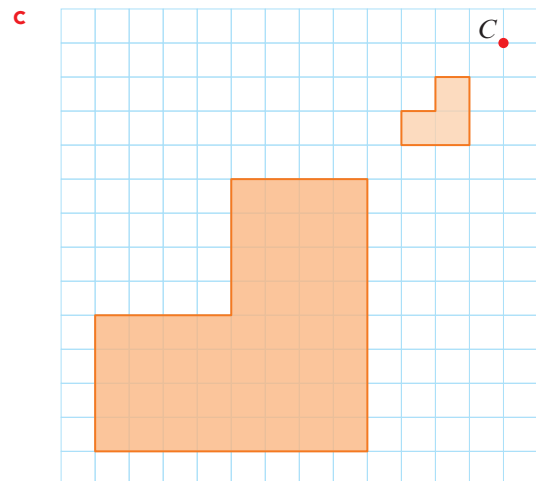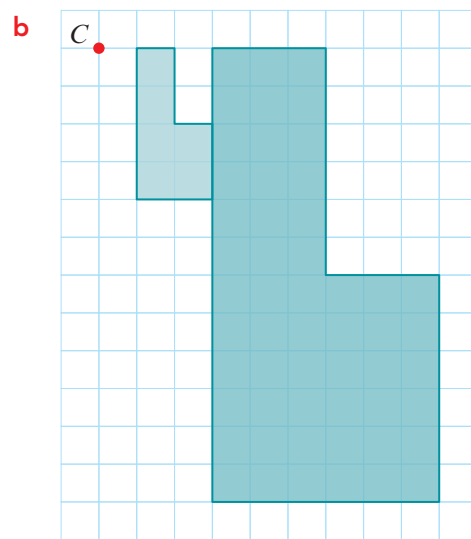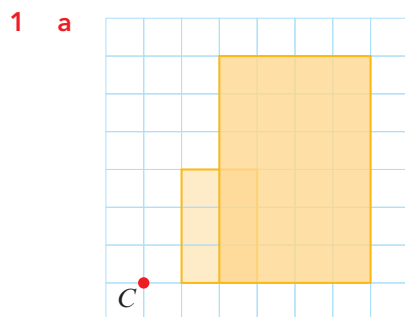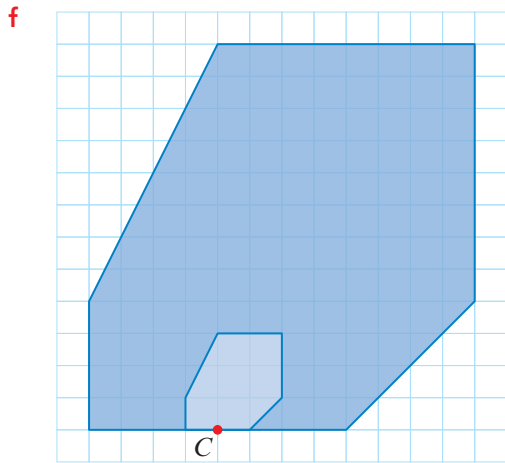
2 a learners' answers

Example: He has used the corner of the image as the centre of enlargement. He should have drawn the image twice as far from the centre of enlargement as the object.

b

3 a

b (4, 4), (10, 4), (4, 7)

4 a i & ii
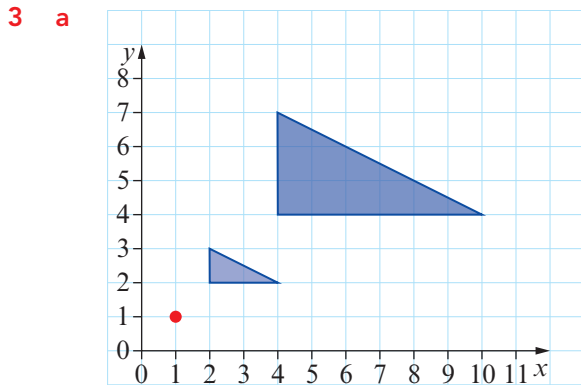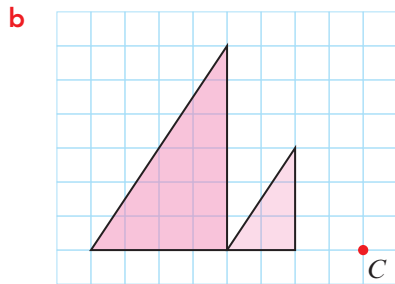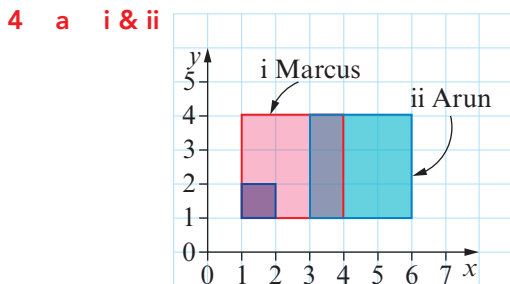
b learners' answers

Example: An invariant point is a point that is in exactly the same position on the image and the object.

c learners' answers

Example: The centre of enlargement must be on the perimeter of the shape.

d learners' answers

Example: The centre of enlargement must be outside the shape.

5 a
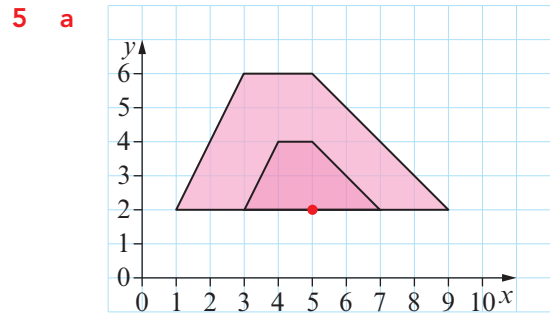
b (1, 2), (9, 2), (5, 6), (3, 6)   **c** (5, 2)

6 a Scale factor 2   **b** Scale factor 3

7 a Scale factor 4

b Zara is correct; learners' answers

8 a
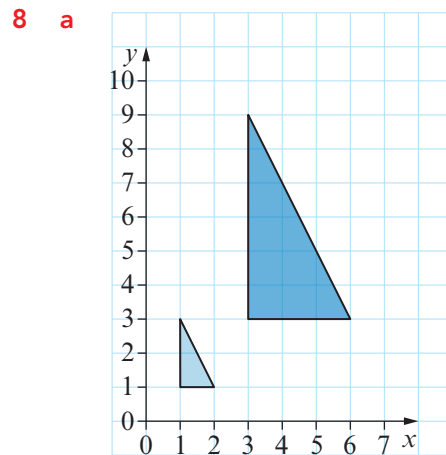
b learners' counter-example

e.g. A triangle with vertices at (1, 1), (2, 1) and (1, 3); enlargement scale factor 2, centre (0, 1).

If Arun is correct, the coordinates of the vertices of the enlargement should be at (2, 2), (4, 2) and (2, 6).
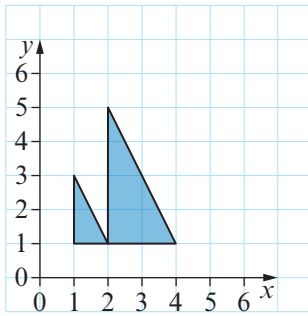
Vertices are at (2, 1), (4, 1) and (2, 5), not (2, 2), (4, 2) and (2, 6)

c (0, 0)

Check your progress

1 correct bearing diagrams drawn for each of these bearings of B from A

a 045° **b** 170°

c 215° **d** 340°

2 a (6, 7) **b** $\left(6\frac{1}{2}, 7\right)$

3 a $\begin{pmatrix} 4 \\ 1 \end{pmatrix}$ **b** $\begin{pmatrix} 1 \\ -4 \end{pmatrix}$

c $\begin{pmatrix} -5 \\ -1 \end{pmatrix}$ **d** $\begin{pmatrix} 0 \\ 4 \end{pmatrix}$

4 ab
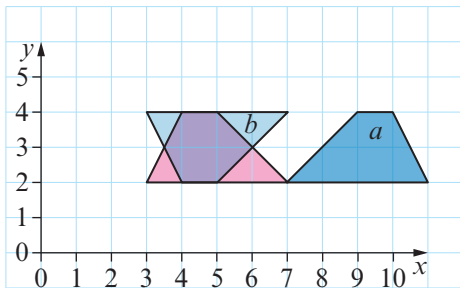
5 a rotation, 90° clockwise, centre (−1, 1)

b rotation, 180°, centre (−2, 0)

6
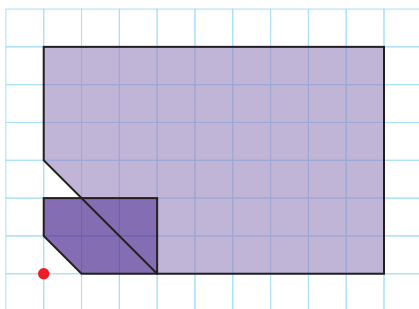
Unit 15

Getting started

1 a 4 **b** 20

c 3 **d** 24

2 a 60 cm² **b** 28 m²

3 a 72 cm³ **b** 108 cm²

Exercise 15.1

1 a T **b** F **c** F

d T **e** F

2 yes, learners' explanations

Example: A kilometre is shorter than a mile, so 35 km is less than 35 miles.

3 a $64 \div 8 = 8$ $8 \times 5 = 40$ miles

b $40 \div 8 = 5$ $5 \times 5 = 25$ miles

c $56 \div 8 = 7$ $7 \times 5 = 35$ miles

4 a $55 \div 5 = 11$ $11 \times 8 = 88$ km

b $20 \div 5 = 4$ $4 \times 8 = 32$ km

c $85 \div 5 = 17$ $17 \times 8 = 136$ km

5 peer discussion

Example of strategies: When converting from smaller units to bigger units (km to miles) there will be fewer of them, so multiply by $\frac{5}{8}$.
When converting from bigger units to smaller units (miles to km) there will be more of them, so multiply by $\frac{8}{5}$.

6 a 15 miles **b** 30 miles

c 60 miles **d** 110 miles

7 a 16 km **b** 160 km

c 200 km **d** 288 km

8 a peer discussion

Example: Easier to change 70 miles to km as 70 is divisible by 5. 107 km is not divisible by 8, so is not easy to change into miles.

b peer discussion

Example: With a calculator it doesn't matter which one you change as the calculator does the maths.

c peer discussion

Example: If the number of km is divisible by 8 or the number of miles is divisible by 5, this will lead to whole number answers which are easier to work out without a calculator.

d **i** 150 km is further (90 miles $\div 5 \times 8 =$ 144 km)

ii 51 miles is further (80 km $\div 8 \times 5 =$ 50 miles)

9 **a** 120 km = 75 miles

b 105 miles = 168 km

c 224 km = 140 miles (or 184 km = 115 miles)

d 115 miles = 184 km (or 140 miles = 224 km)

10 **a** $27\frac{1}{5}$ km or 27.2 km

b $52\frac{4}{5}$ km or 52.8 km

c $86\frac{2}{5}$ km or 86.4 km

d $17\frac{1}{2}$ miles

e $26\frac{1}{4}$ miles

f $46\frac{7}{8}$ miles

11 **a** 1392 km

b Evan is correct. He will get paid $278.40, which is more than $250.

Exercise 15.2

1 **a** $A = bh = 8 \times 4 = 32 \, \text{cm}^2$

b $A = bh = 6 \times 1.5 = 9 \, \text{m}^2$

2 **a** $A = \frac{1}{2} \times (a+b) \times h = \frac{1}{2} \times (6+8) \times 5$

$= \frac{1}{2} \times 14 \times 5 = 7 \times 5$

$= 35 \, \text{cm}^2$

b $A = \frac{1}{2} \times (a+b) \times h = \frac{1}{2} \times (4+12) \times 7$

$= \frac{1}{2} \times 16 \times 7 = 8 \times 7$

$= 56 \, \text{mm}^2$

3 **a** learners' explanations

Example: She has incorrectly used the side length of 5 cm instead of the perpendicular height of 4 cm.

b 28 cm²

4 Yes, they will all get the same answer; learners' explanations

Example: When you multiply numbers together, it doesn't matter in which order you multiply them.

So, $\frac{1}{2} \times (a+b) \times h$ is the same as $(a+b) \times \frac{1}{2} \times h$ (Sofia's method) and $\frac{1}{2} \times h \times (a+b)$ (Marcus's method) and $(a+b) \times h \times \frac{1}{2}$ (Zara's method)

5 learners' answers and explanations. Examples:

a Sofia's method because $a+b=10$ which gives a whole number when you $\div$ by 2.

Also $h=3$, which doesn't give a whole number when you $\div$ by 2.

b Marcus's method because $h=6$ gives a whole number when you $\div$ by 2.

Also $a+b=11$, which doesn't give a whole number when you $\div$ by 2.

c Zara's method because $a+b=5$ and $h=5$. Neither of these gives a whole number when you $\div$ by 2, so it's easier to multiply them together to get 25, then halve the answer.

d Sofia's method or Marcus's method because $a+b=30$ and $h=12$.

Both of these give a whole number when you $\div$ by 2.

6 **a** 25 m²   **b** 51 cm²

c 37.5 mm²

7 **a** learners' explanations

Example: She did not notice that the shapes are measured in different units.

b 702 mm² or 70.2 cm²

8 learners' answers and explanations

Example: Zara is incorrect. If you double the base length of a parallelogram and double the height of the parallelogram, the area of the parallelogram will not be doubled, it will be four times as big.

9 **a** $A = \frac{1}{2} \times (a+b) \times h = \frac{1}{2} \times (2+10) \times 5$

$= \frac{1}{2} \times 12 \times 5 = 30 \, \text{cm}^2$

b 27.83 cm²

10 a b   A and ii, B and iv, C and i, D and v

    c   any shape which has an area of 24.48 cm^2

      For example: parallelogram with length 12.24 cm and height 2 cm

      or rectangle with length 6.12 cm and height 4 cm

11   320 mm or 32 cm

Exercise 15.3

1 a   Area of cross-section $= \frac{1}{2} \times b \times h$

$$= \frac{1}{2} \times 6 \times 8 = 24 \, \text{cm}^2$$

      Volume = area of cross-section × length

$$= 24 \times 10$$
$$= 240 \, \text{cm}^3$$

  b   Area of cross-section $= \frac{1}{2} \times b \times h$

$$= \frac{1}{2} \times 3 \times 4 = 6 \, \text{m}^2$$

      Volume = area of cross-section × length

$$= 6 \times 7$$
$$= 42 \, \text{m}^3$$

2 a   1620 cm^3 **b**   42 m^3

3 a   learners' answers and explanations
Example: The area of a triangle is half the area of a rectangle with the same base and height, so the volume of a triangular prism will be half the volume of a cuboid.

  b, c   learners' answers and explanations

4   learners' explanations

    Example: He has used mixed units. He needs to change 120 mm into cm. Correct answer is 336 cm^3.

5

	Volume
a	8 cm^3
b	1200 mm^3
c	3.15 m^3
d	180 cm^3

6 a   learners' working to show that the volume of the compound prism is 1920 cm^3

      Example:

      Volume of cuboid $= 7 \times 8 \times 20 = 1120 \, \text{cm}^3$

      Volume of prism $= \frac{1}{2} \times 8 \times 10 \times 20 = 800 \, \text{cm}^3$

      Total $= 1120 + 800 = 1920 \, \text{cm}^3$

  b,c   learners' answers and explanations

7 a   81 m^3 **b**   1980 mm^3

8 a   12 cm^2

  b   learners' answers

      Any values for base and height that multiply to give 24.

      Examples:

      Option ①: base = 6 cm and
                       height = 4 cm

      Option ②: base = 8 cm and
                       height = 3 cm

  c   peer discussion – endless possibilities for different values if decimals are used

9 a   7 m

  b   learners' answers and explanations

10 a   450 cm^3

  b   learners' answers; any three numbers that multiply to give 900

      Examples: 9 cm by 10 cm by 10 cm or 18 cm by 5 cm by 10 cm or 9 cm by 5 cm by 20 cm

11   Jan is not correct; learners' explanations. Example: He can only make 8 whole cubes not 9.

    Working: Volume of prism $=$
    $\frac{1}{2} \times 30 \times 12 \times 25 = 4500 \, \text{mm}^3$

    Volume of cube $= 8 \times 8 \times 8 = 512 \, \text{mm}^3$

    Number of cubes $= 4500 \div 512 = 8.789 \ldots$
    $= 8$ whole cubes

Exercise 15.4

1 a

Area of A = 8 × 12
= 96 cm²

Area of B = 6 × 12
= 72 cm²

Area of
C = 10 × 12 = 120 cm²

Area of D = $\frac{1}{2}$ × 6 × 8
= 24 cm²

Area of E = Area of D

Surface area = 96 +
72 + 120 + 2 × 24 =
336 cm²

b

Area of A = 10 × 10
= 100 cm²

Area of B = $\frac{1}{2}$ × 10 × 14
= 70 cm²

Area of C, D and
E = Area of B

Surface area = 100 +
4 × 70 = 380 cm²

2 a i
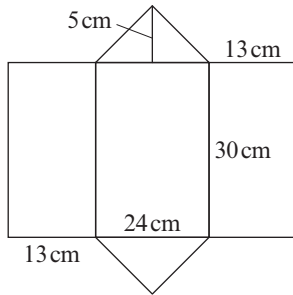
ii   1620 cm²

b i
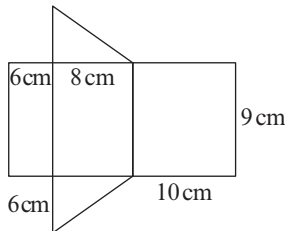
ii   264 cm²

c i
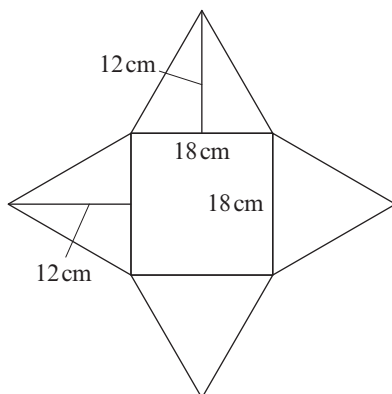
ii   756 cm²

d i
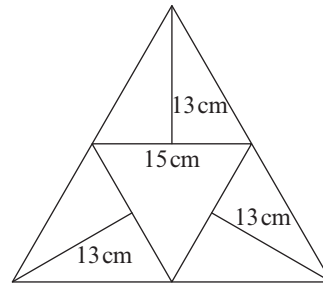
ii   390 cm²

3   The cube has the greater surface area

Surface area of prism = 264 cm²

Surface area of cube = 294 cm²

4   Surface area of pyramid = $\left(\frac{1}{2} \times 4 \times 3.5\right) \times 4$
= 28 m²

Surface area of cuboid = (2 × 2) × 2 +
(2 × 1.5) × 4 = 20 m²

28 − 20 = 8 m², so the surface area of the
triangular-based pyramid is 8 m² more than
the surface area of the cuboid.

5 a x^2

b   Area of triangle = $\frac{1}{2} \times b \times h = \frac{1}{2} \times x \times 2x = x^2$

c   Surface area = $5x^2$

d   Pyramid A $x = 5$ cm, Surface area
= 125 cm²

Pyramid B $x = 7$ cm, Surface area
= 245 cm²

Difference in surface area = 245 − 125
= 120 cm²

e   peer discussion

6   93.9 cm²

7   Surface area = 222 cm²

[$A = 36$ cm², $l = 5$ cm, $h = 6$ cm, $b = 12$ cm,
$x = 11$ cm, $y = 7$ cm]

Check your progress

1 a T   **b** F   **c** T

2 a   70 miles   **b**   328 km

3 a   28 cm²   **b**   160 m²

4   54 cm³

5 **a**

5 cm, 5 cm, 13 cm, 13 cm, 10 cm, 12 cm, 5 cm, 13 cm

b 360 cm²

6 **a**

10 cm, 7 cm
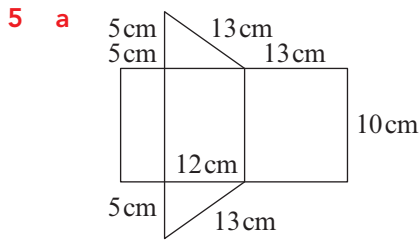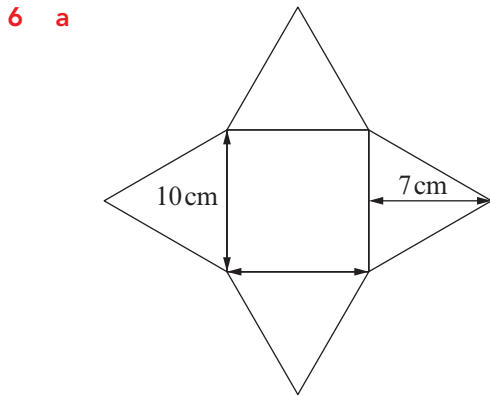
b 240 cm²

Unit 16

Getting started

1 **a** 5 **b** 18 **c** $\frac{3}{18} = \frac{1}{6}$

2 learners' justifications

Examples:

a Pie chart – best way to show proportion

b Line graph – shows the number sold each month and clearly shows how the sales change each month

c Dual bar chart – clearly compares boys and girls

d Frequency diagram – once the data is grouped you can clearly see the spread of the data

3 **a** 25% **b** $\frac{30}{360} = \frac{1}{12}$ **c** 44

4 **a** Mode = $500, median = $505, mean = $550

b learners' choice and reason

Example: the median as it is central to the data. The mean is too high as it is influenced by a small number of higher wages. The mode is too low as there are only 2 wages below the mode.

c $290

Exercise 16.1

1 **a** 8 **b** 7

c 25; added all the frequencies together

2 **a** Completed graph

Number of cups of coffee sold per day
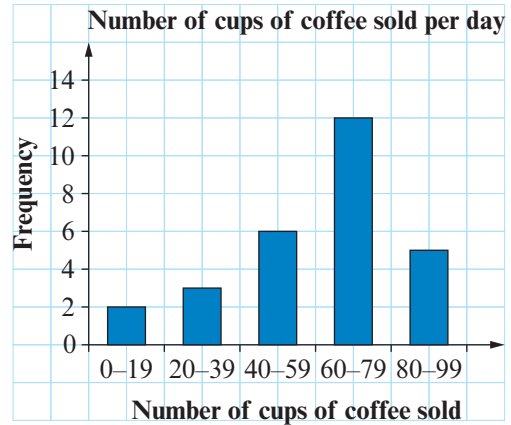
b February. The only month which has only 28 days.

c Not really. It could be 99, but you can't tell from grouped data information; the greatest number of cups of coffee sold could be anywhere from 80 to 99.

3 Learners' tables and diagrams with their own class intervals. One example shown below.

a

Number of text messages	Frequency
0 – 4	4
5 – 9	2
10 – 14	7
15 – 19	9
20 – 24	6

b

Number of text messages sent each day in one month
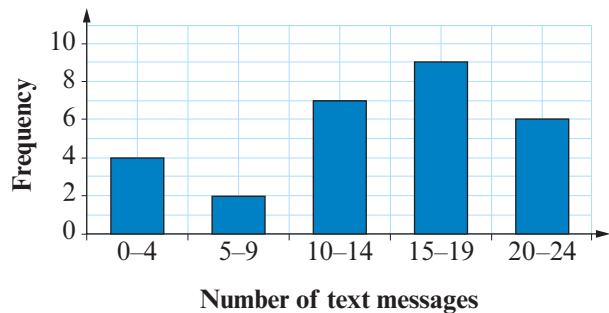
c peer discussion

4 a Zara. The smallest value is 10 and the highest is 39, so she would have 6 classes, which would show a good comparison of the data.

b Marcus's classes are not suitable as there is no data for the first two classes he is using. Arun's classes are not suitable as the classes are too big. There will only be three classes.

c

Number of emails	Frequency
10 – 14	5
15 – 19	6
20 – 24	2
25 – 29	2
30 – 34	8
35 – 39	7

d

Number of emails sent
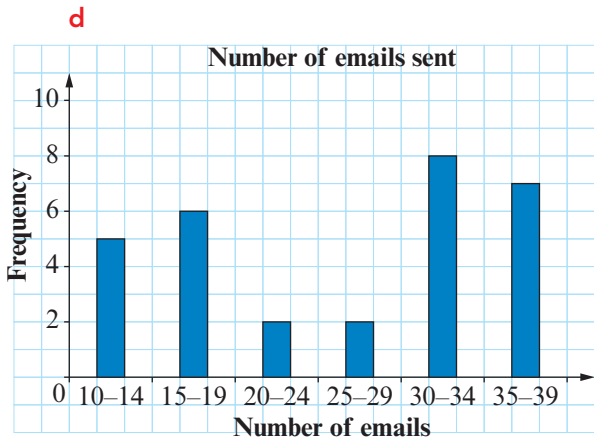
5 learners' answers and discussions

Examples:

a Age from 10 years old, including 10, up to 20 years old, not including 20.

b The data is continuous, not discrete. For example, there wouldn't be a group for anyone aged $19\frac{1}{2}$.

c $30 \leqslant a < 40$

d

Age of members of a choir

e peer discussion

6 a

Speed of cars
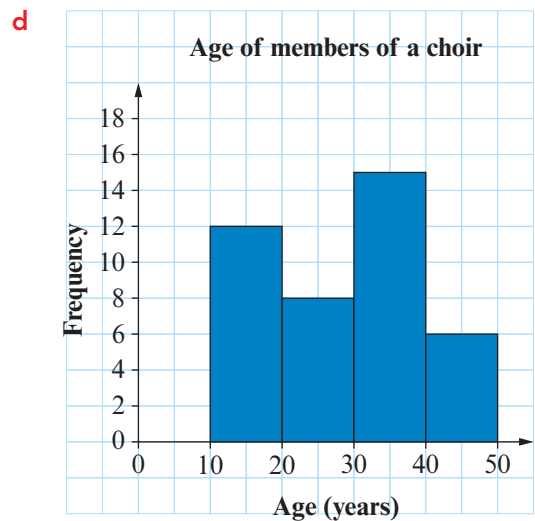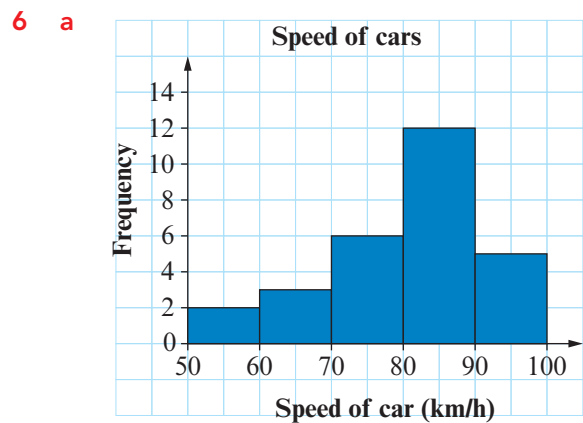
b 17

c No. It could not be 50 km/h as '50 <' means that the speed must be greater than, and not equal to, 50.

7 a

Height, h cm	Frequency
$20 \leqslant h < 25$	3
$25 \leqslant h < 30$	6
$30 \leqslant h < 35$	9
$35 \leqslant h < 40$	2

b

Heights of plants
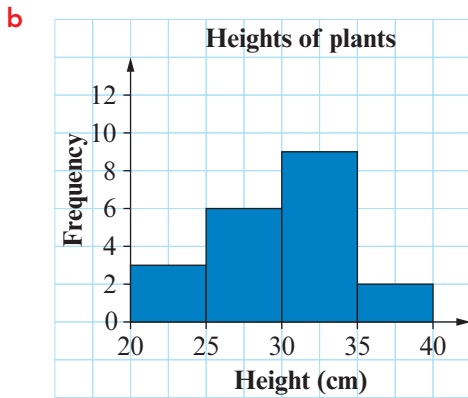
c 17. Add the frequencies of the three bars that show heights that are at least 25 cm.

8 a Learners' sentences, e.g. There are more people in the 0–40 age group in 1960 than in 2010. There is a more even spread across the age groups in 2010 than in 1960.

b Yes he is correct. In 1960, approximately 37 out of 157 people were aged over 40, which is about 25%, but in 2010, approximately 97 out of 162 people were aged over 40, which is almost 60%.

Exercise 16.2

1 a i $1 million   **ii** $1.5 million

b 2008

c 2007 and 2008

d 2010 and 2011

e Example: From 2006 to 2008 profits rise; from 2008 to 2011 profits fall.

2 a i $120 000   **ii** $170 000

b 2008

c 2004 and 2006

d Example: From 2000 to 2004 the value went up slowly. From 2004 to 2006 the value went up faster. From 2006 to 2008 the rate of increase in value was slower, then from 2008 to 2010 the value fell fast.

e i $140 000   **ii** $180 000

3 learners' discussions and answers

Example:

Sofia is correct, you cannot tell from the graph in which year the average price of crude oil was at its highest. This is because the graph only shows the price every 10 years and does not show any intermediate values. It does, however, show that the overall trend is that the price is going up.

4 learners' graphs and answers

Example shown.

Number of people staying in a guest house each month for one year
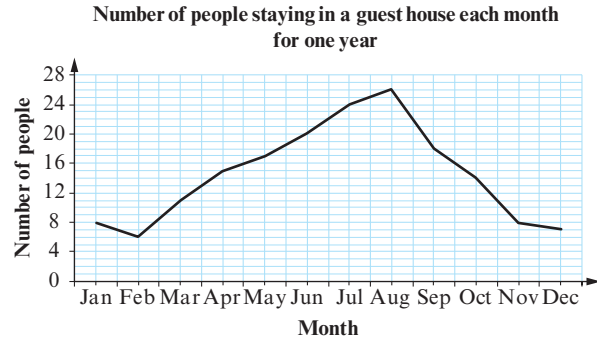
From February to August the number of people staying in the guest house gradually increases. From August to December there is a gradual decrease in the number of people staying in the guest house.

The number of people at the guest house changed the most between August and September.

5 a
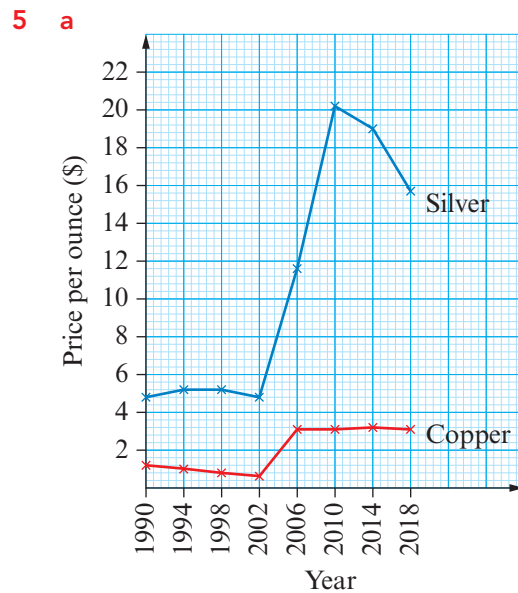
b i T   **ii** F   **iii** F

c i $15.90   **ii** $3.10

d Learners' explanations, e.g. We only know that the prices went up between 2002 and 2006. We do not know exactly when they went up.

6 learners' discussions and answers

Example:

Sofia is correct. If Zara plots every distance she will have more than 150 points to plot. Although this will accurately show the data, to see the overall trend she can plot fewer points. She could plot the distance she cycles every Monday (this would give her approximately 50 plots) or the distance she cycles every second Monday (this would give her approximately 25 plots) or the first distance she cycles every month (this would give her 12 plots) or the monthly average (this would give her 12 plots).

7 learners' answers

Examples:

a i The level of stock is falling at a steady rate so sales are steady.

ii The level of stock is falling at a reducing rate and much more slowly than that of the Scarlets. Sales are slow and declining.

b No. If the trend continues, they will sell out half way through the week.

c Yes. If the trend continues, they will only sell 1 or 2 shirts and they have 4 in stock.

8 a Learners' descriptions. Example: The number of hotel rooms booked increases from Spring to Summer and then from Summer to Autumn. The number then decreases from Autumn to Winter and from Winter to the following Spring.

b Yes. Learners' explanations. Example: In all three years the numbers increase from Spring to Summer to Autumn, then decrease from Autumn to Winter.

c Learners' descriptions. Example: Overall the number of rooms that are booked each year is increasing.

d Learners' predictions. Example: 8400 rooms. (Accept answer in the range 8100 to 9000.)

e Learners' explanations. Example: It is a prediction into the future, so it may not actually happen.

Exercise 16.3

1 a 15 **b** 45 minutes

c 5

d i 45 minutes **ii** 56 minutes

iii 22 minutes

2 learners' answers and discussions

Examples:

a i February (The only month which has 28 days)

ii 158 **iii** 147

b The key is missing from the diagram. You can guess the key as the diagram is about the number of cups of coffee sold in a cafe, so the numbers cannot be decimal numbers.

3 a Key: 10 | 1 means 101 kB

10	1 3 8 9
11	0 7 7 7
12	5 5 8
13	0 0 1 5 9 9
14	0 5 8
15	1 2 4 5 8
16	0 2 5 6 8

b 10

c mode = 117 kB, median = 137 kB. Median better represents this data as it is in the middle of the data. The mode is very close to the smallest value.

d Greg is incorrect. The range is 67 kB. ($168 - 101 = 67$)

4 learners' answers and discussions

Example:

Opaline's method is incorrect. It would work if the number of values in each line of the diagram was the same. If the number of values in each line of the diagram is different, then you need to find the total of all the values and divide by the number of values as usual.

You could find the total of each line first, then find the total of these totals, then divide by the number of numbers – this is possibly an easier method to use without a calculator.

5 **a** **i** mode = 24   **ii** median = 8

    iii mean = 12

  b mean with reason

    Example: The mean is the best to use as the mode is the same as the greatest value and the median is too low so neither of these averages represents the majority of the data.

6 **a** 44%

  b $\frac{5}{25} = \frac{1}{5}$ **c** 20

Exercise 16.4

1 **a** Seals

  b Dolphins: $\frac{150}{360} \times 120 = 50$ girls

    Seals: $\frac{240}{360} \times 72 = 48$ girls

    The Dolphins swimming club has the greater number of girls.

2 **a** **i** $\frac{1}{2}$ **ii** $\frac{1}{4}$

  b **i** halved

    ii stayed the same

    iii tripled

  c 2009: $\frac{1}{2} \times 12\,000 = \$6000$

    2019: $\frac{1}{4} \times 24\,000 = \$6000$

  d 2009: $\frac{45}{360} = \frac{1}{8}$, $\frac{1}{8} \times 12\,000 = \$1500$

    2019: $\frac{135}{360} = \frac{3}{8}$, $\frac{3}{8} \times 24\,000 = \$9000$
    $6 \times \$1500 = \$9000$

  e \$4500

3 **a** 120   **b** 135

  c No. Men = 180, women = 200.

  d More women than men took part in the survey. When the angles in the pie charts are the same, the women's sector must represent a greater number than the men's sector.

4 Castlehill School. Castlehill School = 160, Riverside School = 154.

5 **a** **i** 1200   **ii** 1800

    iii 150

  b,c learners' methods and explanations

6 **a** 32   **b** 288

7 **a**

Amount of rice sold in Shop A		
Type of rice	Degrees in pie chart	Kilograms sold
black	30°	6 kg
brown	120°	24 kg
red	60°	12 kg
white	150°	30 kg
Total:	360°	72 kg

  b

| Amount of rice sold in Shop B		
Type of rice	Degrees in pie chart	Kilograms sold
black	20°	8 kg
brown	180°	72 kg
red	30°	12 kg
white	130°	52 kg
Total:	360°	144 kg

  c learners' explanations

    Example: In shop A the angle for red rice is double the angle for red rice in shop B. If the amount sold is the same, this means that the total amount of rice sold in Shop A is half the total amount of rice sold in Shop B.

Exercise 16.5

1 **a** Dual bar chart – probably the easiest way to compare the data. Pie chart or compound bar chart are also good ways to compare proportions (percentages).

  b Scatter graph – you can plot two sets of data for each horse and see if there is any connection between age and height.

  c Stem-and-leaf diagram – you can see the actual scores, as well as the grouping of the scores.

  d Time series graph – you can clearly see how the mass of the baby chimpanzee changes over time.

2 a
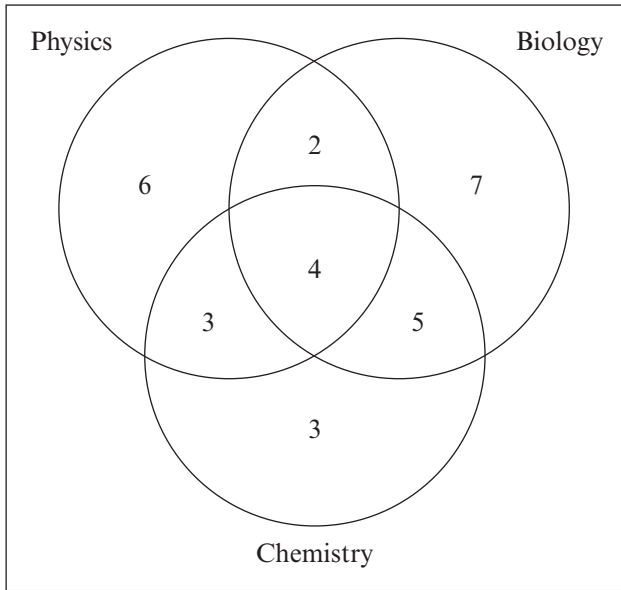
b Example: Clearly shows the number of learners taking the different combinations of science subjects.

c Example: More learners took Biology than any other science subject.

3 a
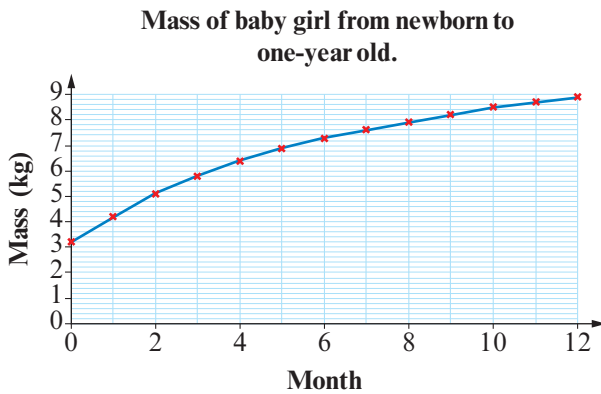
b Example: Clearly shows how the mass of the baby changes over time.

c Example: The mass of the baby increased more in the first half of the year than in the second half of the year.

4 peer discussion

a i Example: amounts of the individual ingredients

ii Example: the proportions

b i Example: the proportions

ii Example: amounts of the individual ingredients
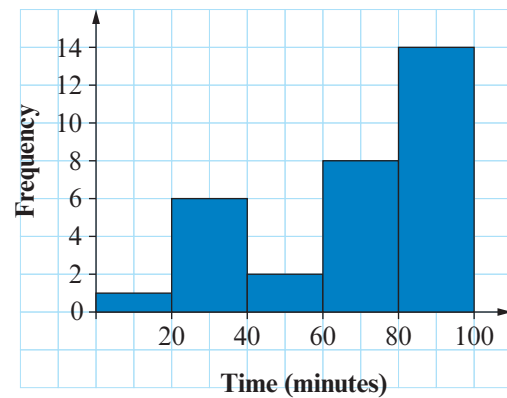
c i compound bar chart

ii pie chart

5 a Key: 0 | 5 represents 5 pages

0	5 7 8 9
1	0 1 1 2 2 4 6 7 8 9
2	0 2 2 4 5 7 7 7
3	0 2 3 4 5 9

b Example: It shows the individual data as well as the overall spread of the data.

c Example: It shows that Daylen only read fewer than 10 pages on four days.
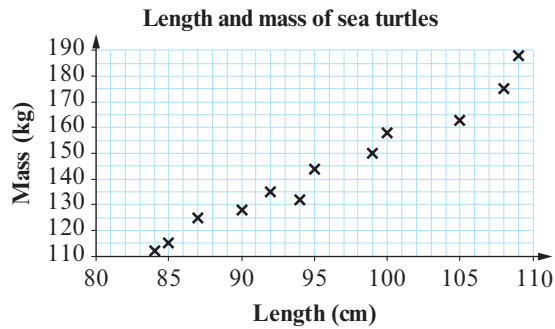
d i 27 pages   **ii** 19.5 pages

iii 34 pages

6 a graph drawn

b Example: Best way to show continuous grouped data.

c Example: Zara spent just under half the evenings in this month doing homework for 80 minutes or more.

7 a

Length and mass of sea turtles

b Example: You can plot two sets of data for each turtle and see if there is any connection between length and mass.

c Example: The longer a turtle the heavier it is.

Exercise 16.6

1 a Spain: 1.14, Brazil: 1.8

b Brazil scored more goals, on average, per match.

c Spain: 2, Brazil: 3

d Brazil's scores were more varied.

2 a i Group A: 73, 77, 80, 84, 89   Group B: 67, 69, 72, 75, 77, 82, 85

ii Group A: 80 cm   Group B: 75 cm

iii Group A: 16 cm   Group B: 18 cm

b Group A is taller, on average.

c Group A's heights are less varied.

3 a i Madrid: 32, 34, 35, 36, 36, 37, 38   Cartagena: 28, 29, 30, 30, 30, 32, 33

ii Madrid: 36 °C   Cartagena: 30 °C

iii Madrid: 6 °C   Cartagena: 5 °C

b Madrid is hotter, on average.

c Madrid's temperatures are more varied.

4 The girls were heavier on average.

Mean mass of baby boys was 3.2 kg. Mean mass of baby girls was 3.4 kg.

5 a

	Mean	Median	Mode	Range
Maths	79.8	77.5	76	16
Science	77.5	78	87	25

b learners' decision; could use median or mean. Mode is not suitable as it's the second lowest score in maths and the highest score in science.

c Maths had more consistent scores – 16 is less than 25

d,e peer discussion

6 a

	Mean	Median	Range
First	28.20 °C	29 °C	12 °C
Second	30.25 °C	30 °C	5 °C

b i False – the median and the mean for the second experiment are higher than the median and the mean for the first experiment

ii True – the range for the first experiment is greater than the range for the second experiment

c No – all the temperatures recorded in the first experiment are different so there is no mode. The second experiment has no one temperature that occurred the most often.

7 a 0   **b** 5

c to g learners' results, answers and discussions

8 a

|	Mean	Median	Mode
Team A	2.8	3	5
Team B	2.95	3	1

Marcus is correct if you use the mode, Zara is correct if you use the mean, Arun is correct if you use the median.

b learners' decision; could use median or mean. Mode is not suitable as it's the highest score for team A and the second lowest score for team B.

Check your progress

1 a

Age, a years	Frequency
$10 \leqslant a < 15$	4
$15 \leqslant a < 20$	3
$20 \leqslant a < 25$	8
$25 \leqslant a < 30$	5

b graph drawn

Ages of members of bowling club
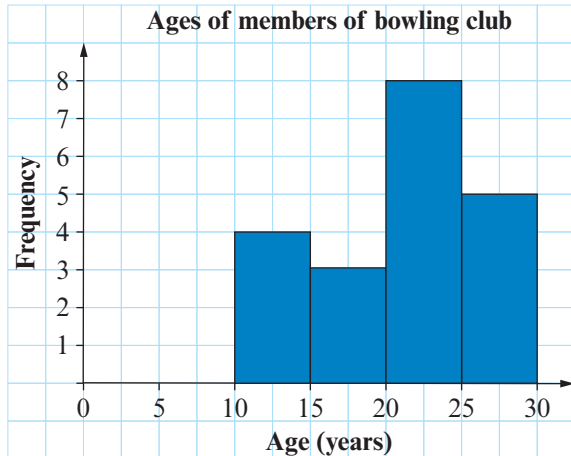
c 13; added the last two frequencies in the table, $8 + 5 = 13$

2 a Key: $0 \mid 8$ means a score of 8

```
0 | 8 9
1 | 2 8 8
2 | 4 7 8 8 8 8 9
3 | 4 6 6 7 8 9 9
4 | 3 5 6 9
5 | 0 0
```

b 48%   **c** $\frac{6}{25}$ **d** 20

e i Mode is 28

ii Median is 34, mean is 32

3 Haywood School

Haywood School: 12% of $1200 = 144$

Ryefield School: 18% of $700 = 126$

4 a

	Mean	Median	Mode	Range
Art	12.2	11	16	10
Music	13.5	14	6	14

b Music; learners' decision; median or mean. Mode is not suitable as it's the second highest score for art and the lowest score for music.

c Art: The range is lower for art than for music.